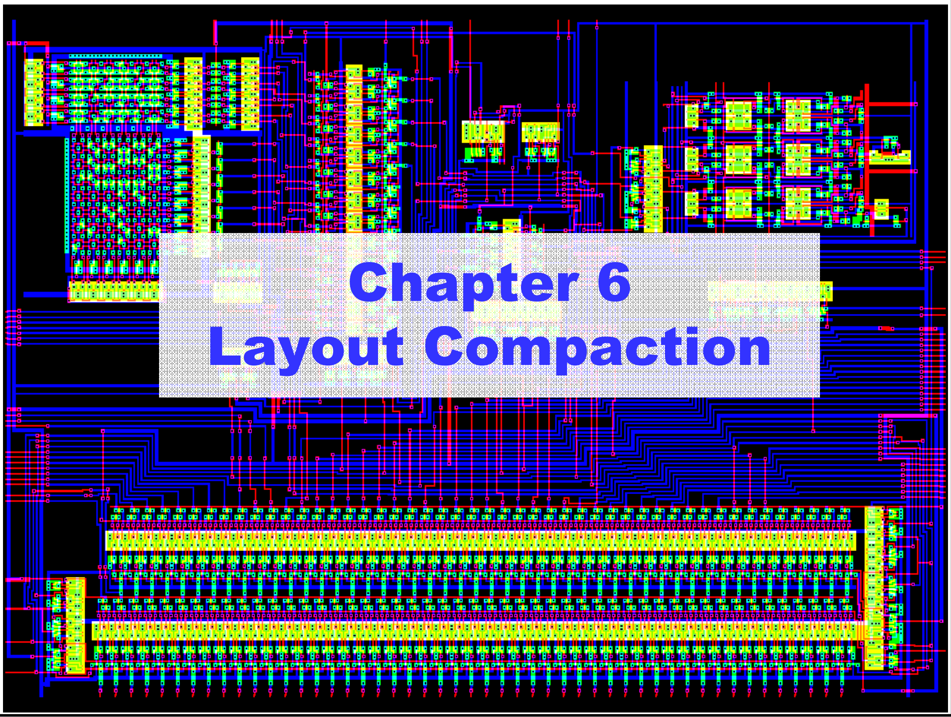


Design Rules

- The fabrication process will suffer from tolerances
- Chip features will have a practical minimum size to allow them to be fabricated reliably enough (with high enough yield)
- This is captured into a set of precise Design Rules
- Modern processes have terribly complex set of design rules as a compromise between flexibility and manufacturability

http://en.wikipedia.org/wiki/Design_rule_checking

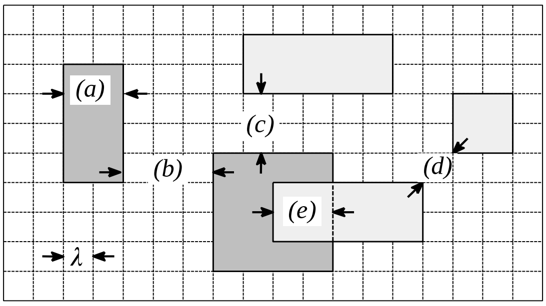
Design Rules

Design rules: restrictions on the mask patterns to increase the probability of successful fabrication.
Compromise between

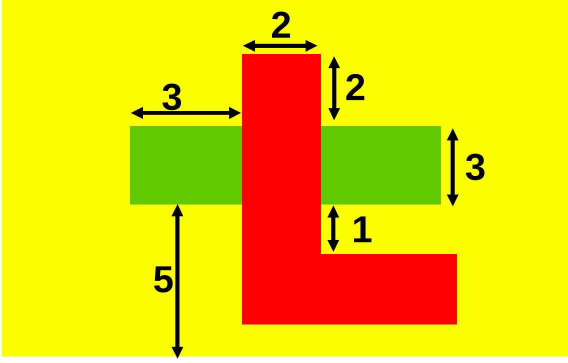
- Density
- Yield
- Ease of use

Patterns and design rules are expressed in either nanometers or integer multiples of 'gridsize' λ. The types of the most common design rules:

- minimum-width rules (a)
- minimum-separation rules (b, c, d)
- minimum-overlap rules (e)

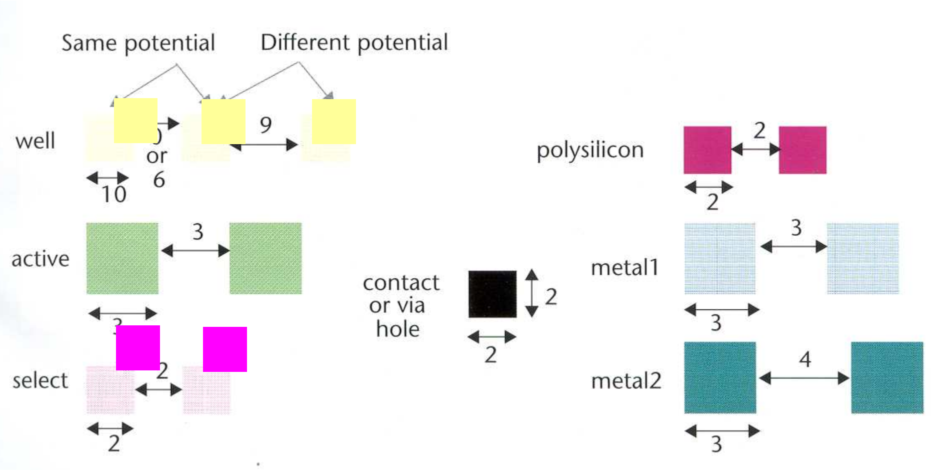


λ-based Transistor Rules



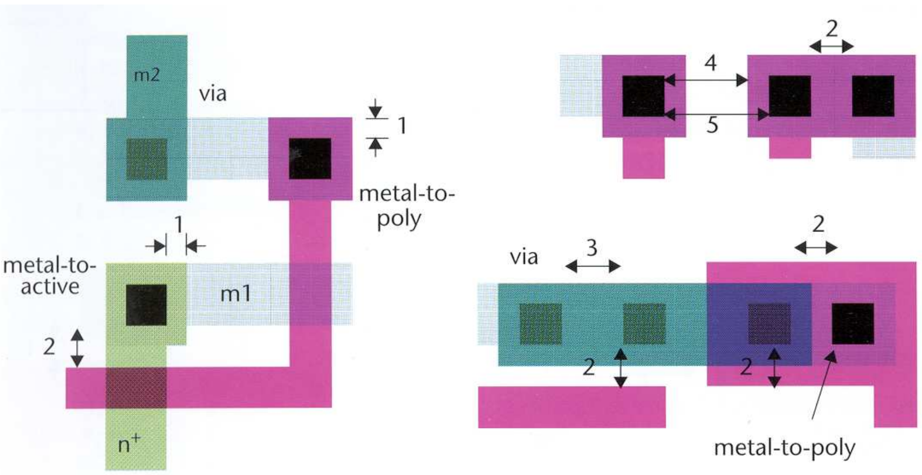
- Illustration only

More DR Examples (Intra-layer)



Intra-layer design rules: minimum dimensions and spacings

Vias & Contacts.



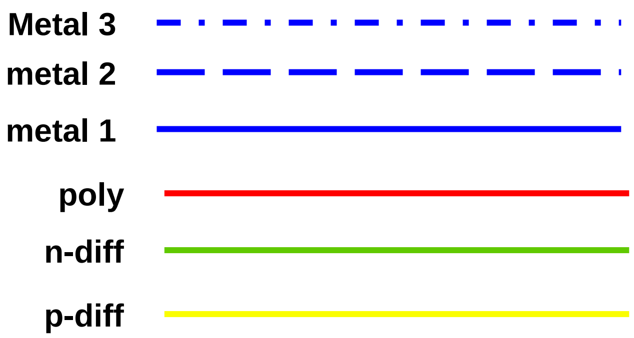
Symbolic Layout

A layout is symbolic when not all mask patterns have full specification:

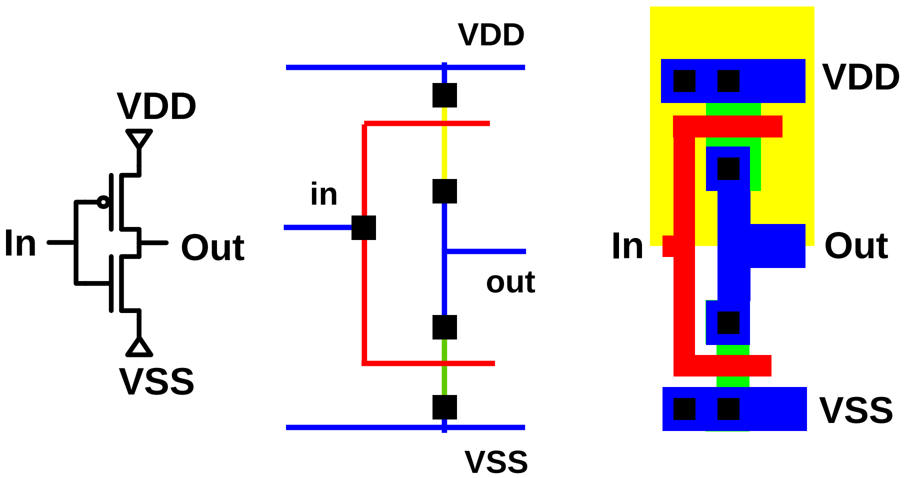
- Single symbols are used to represent elements located in several layers, e.g. transistors, contact cuts.
- The length, width or layer of a wire or other layout element might be left unspecified.
- Mask layers not directly related to the functionality of the circuit do not need to be specified, e.g. n-well, p-well.

Symbolic Layout / Stick diagrams

- A stick diagram is a cartoon of a layout.
- Does show components/vias but only relative placement.
- Does not show exact placement, transistor sizes, wire lengths, wire widths, tub boundaries, some special components.



Invertor Schematic, Stick Diagram, Layout.



Compaction and its Applications

A compaction program or compactor generates layout at the mask level. It attempts to make the layout as dense as possible.

Applications of compaction:

- Area minimization: removing redundant space in layout at the mask level.
- Layout compilation: generation of mask-level layout from symbolic layout.
- Redesign: automatic removal of design-rule violations.
- Rescaling: converting mask-level layout from one technology to another.

Aspects of Compaction

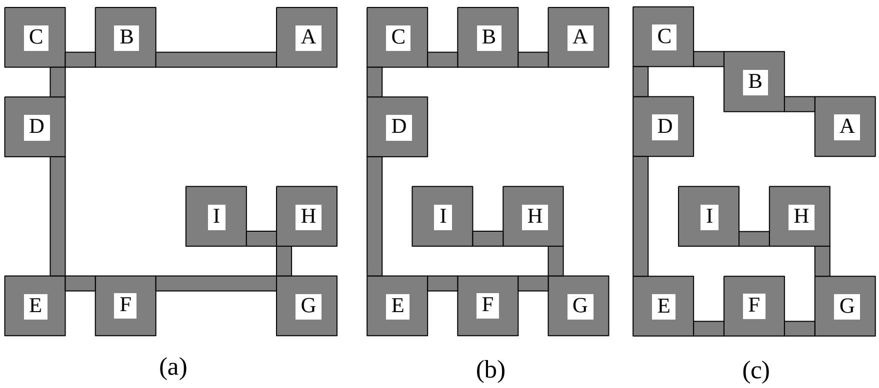
Dimension:

- 1-dimensional (1D): layout elements only move or shrink in one dimension (x or y). Often sequentially performed first in the x-dimension and then in the y-dimension (or vice versa).
- 2-dimensional (2D): layout elements move and shrink simultaneously in two dimensions.

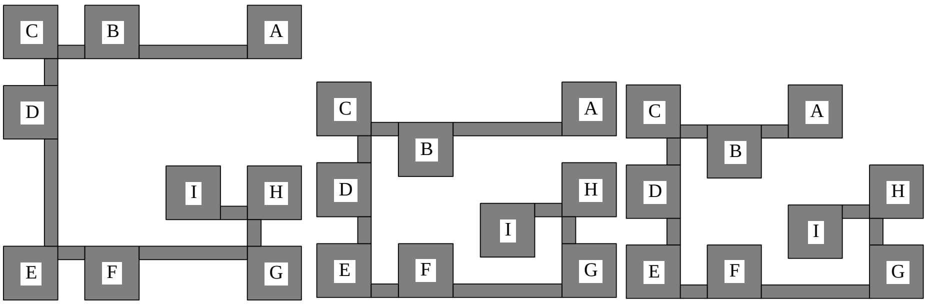
Complexity:

- 1D-compaction can be done efficiently; 2D-compaction is NP-hard.

1D Compaction: X Followed by Y

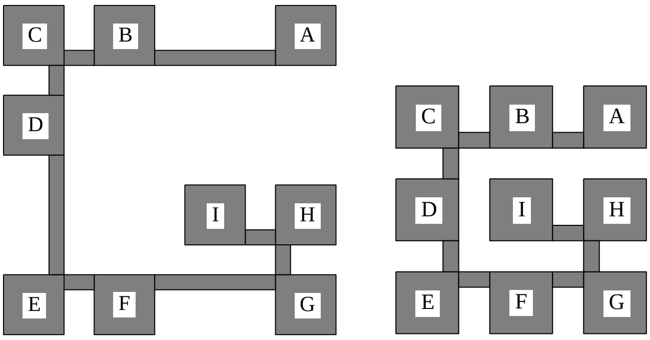


1D Compaction: X Followed by Y



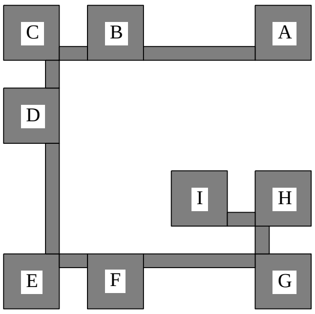
(a) (d) (e)

2D Compaction



(a) (f)

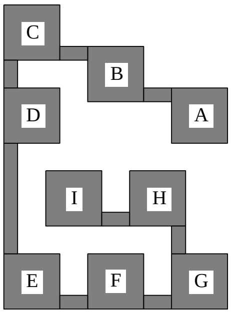
Comparison



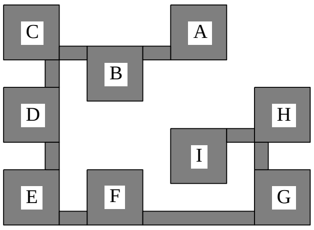
orig

(a)

x then y



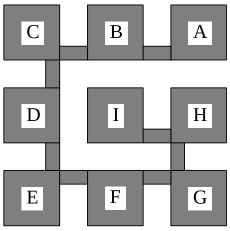
(c)



y then x

(e)

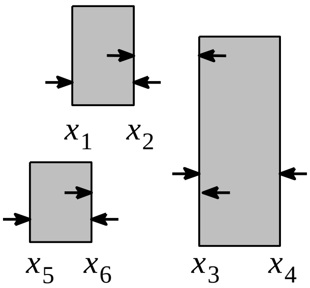
2D



(f)

Inequalities for Distance Constraints

Minimum-distance design rules can be expressed as inequalities.



For example:

$$x_2 - x_1 \geq a$$

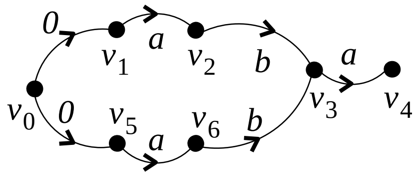
$$x_3 - x_2 \geq b$$

$$x_3 - x_6 \geq b$$

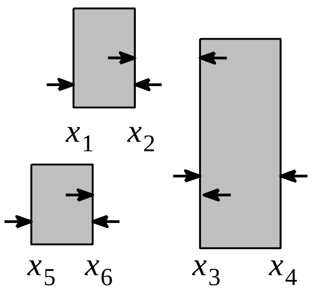
The Constraint Graph

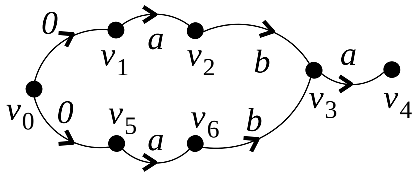
The inequalities can be used to construct a **constraint graph G(V, E)**:

- There is a **vertex** v_i for each **variable** x_i.
- For each inequality of the form $x_j - x_i \geq d_{ij}$, there is an **edge** (v_i, v_j) with **weight** d_{ij}.
- There is an extra **source vertex**, v_0; it is located at $x = 0$; all other vertices are at its right.
- If all the inequalities express minimum-distance constraints, the graph is acyclic. It is a **DAG**, a **directed acyclic graph**.



The Constraint Graph





Example rules

- All widths ≥ a
- All spacings ≥ b

Maximum-Distance Constraints

Sometimes the distance of layout elements is bounded by a maximum, e.g. when the user wants a maximum wire width.

- A **maximum distance constraint** gives an inequality of the form:

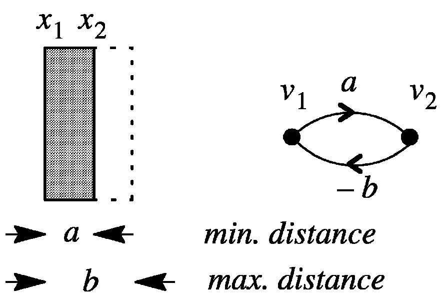
$$x_j - x_i \le c_{ij}$$

$$\Leftrightarrow$$

$$x_i - x_j \ge -c_{ij}.$$

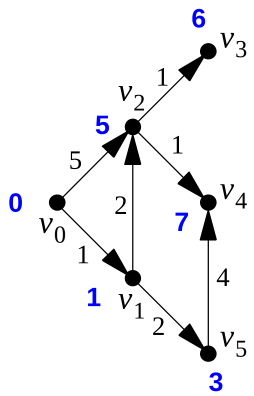
- Consequence for the constraint graph: **backward edge** (v_j, v_i) with weight $d_{ji} = -c_{ij}$.

The graph is not acyclic anymore.



Longest Path Problem

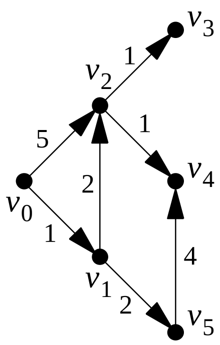
- Given graph with weighted edges, assign positions to nodes that is consistent with all the edges



Longest-Path Algorithm for DAGs

longest-path(G)
{
  for ($i \leftarrow 1$; $i \leq n$; $i \leftarrow i + 1$)
    $p_i \leftarrow$ "in-degree of v_i";
  $Q \leftarrow \{v_0\}$;
  while ($Q \neq \emptyset$) {
    $v_i \leftarrow$ "any element from Q";
    $Q \leftarrow Q \setminus \{v_i\}$;
    for each v_j "such that" $(v_i, v_j) \in E$ { // for each outgoing edge
      $x_j \leftarrow \max(x_j, x_i + d_{ij})$; // set max distance for x_j
      $p_j \leftarrow p_j - 1$;
      if ($p_j \leq 0$) // if all incoming edges have been processed
        $Q \leftarrow Q \cup \{v_j\}$; // can process its outgoing edges
    }
  }
}

main ()
{
  for ($i \leftarrow 0$; $i \leq n$; $i \leftarrow i + 1$)
    $x_i \leftarrow 0$;
  longest-path(G);
}



while ($Q \neq \emptyset$) {
  $v_i \leftarrow$ "any element from Q";
  $Q \leftarrow Q \setminus \{v_i\}$;
  for each v_j "such that" $(v_i, v_j) \in E$ {
    $x_j \leftarrow \max(x_j, x_i + d_{ij})$;
    $p_j \leftarrow p_j - 1$;
    if ($p_j \leq 0$)
      $Q \leftarrow Q \cup \{v_j\}$;
  }
}

Q	p_1	p_2	p_3	p_4	p_5	x_1	x_2	x_3	x_4	x_5
"notinitialized"	**1**	**2**	**1**	**2**	**1**	**0**	**0**	**0**	**0**	**0**
$\{v_0\}$	**0**	**1**	**1**	**2**	**1**	**1**	**5**	**0**	**0**	**0**
$\{v_1\}$	**0**	**0**	**1**	**2**	**0**	**1**	**5**	**0**	**0**	**3**
$\{v_2, v_5\}$	**0**	**0**	**0**	**1**	**0**	**1**	**5**	**6**	**6**	**3**
$\{v_3, v_5\}$	**0**	**0**	**0**	**1**	**0**	**1**	**5**	**6**	**6**	**3**
$\{v_5\}$	**0**	**0**	**0**	**0**	**0**	**1**	**5**	**6**	**7**	**3**
$\{v_4\}$	**0**	**0**	**0**	**0**	**0**	**1**	**5**	**6**	**7**	**3**

Longest-Path Algorithm for DAGs

```
longest-path(G)
{
  for (i ← 1; i ≤ n; i ← i + 1)
    p_i ← "in-degree of v_i";
  Q ← {v_0};
  while (Q ≠ Ø) {
    v_i ← "any element from Q";
    Q ← Q \{v_i};
    for each v_j "such that" (v_i, v_j) ∈ E {
      x_j ← max(x_j, x_i + d_ij);
      p_j ← p_j − 1;
      if (p_j ≤ 0)
        Q ← Q ∪ {v_j};
    }
  }
}
```

```
main ()
{
  for (i ← 0; i ≤ n; i ← i + 1)
    x_i ← 0;
  longest-path(G);
}
```

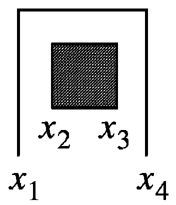
Q: What is the worst-case time complexity of this algorithm?
A: $O(|E|)$

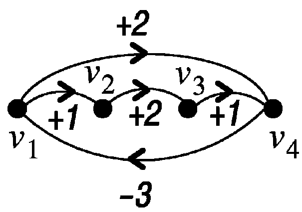
Longest-Paths in Cyclic Graphs

Constraint-graph compaction with maximum-distance constraints requires solving the longest-path problem in cyclic graphs.

Two cases are distinguished:

- **There are positive cycles ⇒ the problem is NP-hard; however, a positive cycle corresponds to a set of conflicting constraints. The best to be done is to detect the cycles.**





- **All cycles are negative: polynomial-time algorithms exist.**

The Liao-Wong Algorithm (1)

Main ideas:

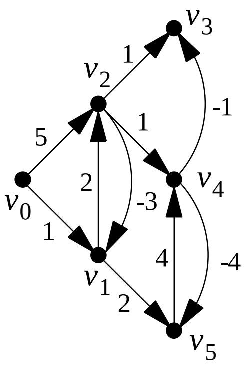
- Split the edge set E of the constraint graph into two subsets:
  - forward edges E_f:related to minimum-distance constraints,
  - backward edges E_b:related to maximum-distance constraints.
- The graph $G(V, E_f)$ is acyclic; the minimum distance for each vertex can be computed with the procedure ''longest-path''.
- Repeat until convergence:
  - update minimum distances by processing the edges from E_b.
  - call ''longest-path'' for $G(V, E_f)$.

The Liao-Wong Algorithm (2)

```
count ← 0;
for (i ← 1; i ≤ n; i ← i + 1)
  x_i ← −∞;
x_0 ← 0;

do { flag ← 0;
     longest-path( G_f );
     for each (v_i, v_j) ∈ E_b
       if (x_j < x_i + d_ij) {
         x_j ← x_i + d_ij;   // backward edge reduces distance
         flag ← 1;           // not yet converged
       }
     count ← count +1;
     if (count > |E_b| && flag)
       error("positi ve cycle")
}
while (flag);          // while not converged
```

The Liao-Wong Algorithm (3).



Step	x_1	x_2	x_3	x_4	x_5
Initialize	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$
Forward1	1	5	6	7	3
Backward1	2	5	6	7	3
Forward2	2	5	6	8	4
Backward2	2	5	7	8	4
Forward3	2	5	7	8	4
Backward3	2	5	7	8	4

- After first forward iteration, the max-3 constraint between v_2 and v_1 is violated
- Corrected after first backward iteration
- But then v_4 and v_5 are too close to v_1
- Etc.

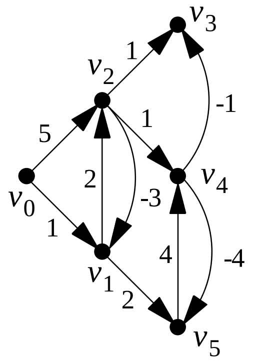
Q: What is the worst-case time complexity of this algorithm?
A: $O(|E_b| \times |E|)$.

The Bellman-Ford Algorithm (1)

http://en.wikipedia.org/wiki/Bellman-ford

Repeated wavefront propagation

```
for (i ← 1; i ≤ n; i ← i + 1)
  x_i ← −∞;
x_0 ← 0;
count ← 0;
S_1 ← {v_0};          // current wavefront
S_2 ← Ø;              // wavefront for next iter
while (count ≤ n && S_1 ≠ Ø) {
   for each v_i ∈ S_1
     for each v_j "such that" (v_i, v_j) ∈ E
       if (x_j < x_i + d_ij) {
          x_j ← x_i + d_ij;
          S_2 ← S_2 ∪ {v_j}
       }
   S_1 ← S_2;
   S_2 ← Ø;             // swap wavefront
   count ← count + 1;   // counter to detect positive loops
}
if (count > n)
  error("positi ve cycle");
```



while (count $\leq n$ && $S_1 \neq \emptyset$) {
  for each $v_i \in S_1$
    for each v_j "such that" $(v_i, v_j) \in E$
      if $(x_j < x_i + d_{ij})$ {
        $x_j \leftarrow x_i + d_{ij}$;
        $S_2 \leftarrow S_2 \cup \{v_j\}$
      }
  $S_1 \leftarrow S_2$;
  $S_2 \leftarrow \emptyset$;
  count $\leftarrow$ count + 1;
}

$x_1 \leftarrow x_2 + (-3)$

S_1	x_1	x_2	x_3	x_4	x_5
"notinitialized"	$-\infty$	$-\infty$	$-\infty$	$-\infty$	$-\infty$
$\{v_0\}$	1	5	$-\infty$	$-\infty$	$-\infty$
$\{v_1, v_2\}$	2	5	6	6	3
$\{v_1, v_3, v_4, v_5\}$	2	5	6	7	4
$\{v_4, v_5\}$	2	5	6	8	4
$\{v_4\}$	2	5	7	8	4
$\{v_3\}$	2	5	7	8	4

Longest Path vs Bellman-Ford

Kernel of algorithms

while $(Q \neq \emptyset)$ {
  $v_i \leftarrow$ "any element from Q";
  $Q \leftarrow Q \setminus \{v_i\}$;
  for each v_j "such that" $(v_i, v_j) \in E$ {
    $x_j \leftarrow \max(x_j, x_i + d_{ij})$;
    $p_j \leftarrow p_j - 1$;
    if $(p_j \leq 0)$
      $Q \leftarrow Q \cup \{v_j\}$;
  }
}

Longest Path

while (count $\leq n$ && $S_1 \neq \emptyset$) {
  for each $v_i \in S_1$
    for each v_j "such that" $(v_i, v_j) \in E$
      if $(x_j < x_i + d_{ij})$ {
        $x_j \leftarrow x_i + d_{ij}$;
        $S_2 \leftarrow S_2 \cup \{v_j\}$
      }
  $S_1 \leftarrow S_2$;
  $S_2 \leftarrow \emptyset$;
  count $\leftarrow$ count + 1;
}

Bellman-Ford

Worst-case: $O(n \times |E|) = O(n^3)$.

Average-case: $O(n^{1.5})$. (Schiele)

Longest and Shortest Paths

- Longest paths become shortest paths and vice versa when edge weights are multiplied by –1.
- Situation in DAGs: both the longest and shortest path problems can be solved in linear time.
- Situation in cyclic directed graphs:

  + All weights are positive: shortest-path problem in P (Dijkstra), longest-path problem is NP-complete.

  + All weights are negative: longest-path problem in P (Dijkstra), shortest-path problem is NP-complete.

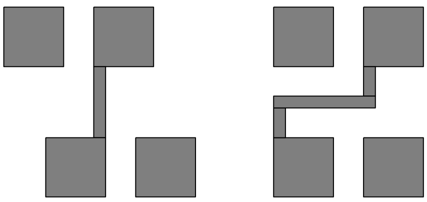
  + No positive cycles: longest-path problem is in P

  + No negative cycles: shortest-path problem is in P.

  + Otherwise: problem is NP-complete.

Remarks Constraint-Graph Compaction

- The algorithms mentioned only compute the left-most position for each layout element. All elements outside the critical paths also have a right-most position. It is interesting to find the best position within this interval with respect to some cost function.
- The quality of the layout can further be improved by automatic jog insertion.

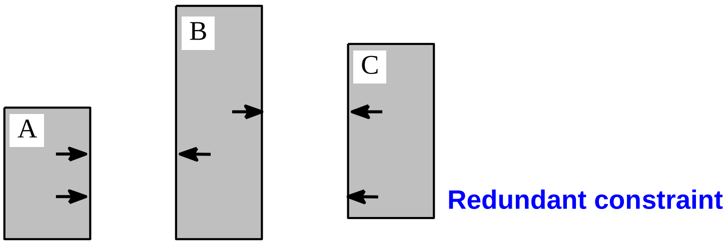


(a) (b)

- A method to reduce complexity is hierarchical compaction.

Constraint Generation

- The constraint graph is not directly available after layout design. It must be computed.
- The set of constraints should be irredundant (why?) and generated efficiently.



- Doenhardt and Lengauer have proposed a method for irredundant constraint generation with complexity $O(n \log n)$.

Virtual Grid Compaction

Features:

- 1D method.
- All layout elements are associated to horizontal and vertical grid lines.
- Initially the distance between grid lines is unspecified.
- The distance between two subsequent grid lines is computed by taking the maximum separation imposed by pairs of elements on different grid lines.
- Disadvantage: rigid as elements initially located on one line always remain aligned.