Partial Inductance Extraction with an Exponentially Damped Potential Compared to Virtual Screening

A. J. Dammers[1] and N. P. van der Meijs[2]
[1] *Netherlands Institute for Metals Research and Delft University of Technology*
Department of Materials Science, Rotterdamseweg 137, 2628 AL Delft (NL)
[2] *Delft Institute of Microelectronics and Submicron Technology*
Department of Electrical Engineering, Circuits and Systems Section
Delft University of Technology, P.O. Box 5031, 2600 GA Delft (NL)

Abstract

In order to improve the efficiency of the PEEC method for large scale interconnect inductance extraction, enhancing robustness under matrix truncation is of vital importance. We establish the relationship between the virtual screening approach of Dammers and van der Meijs (ICCAD'99) and the damped potential method of Beattie and Pileggi (DAC'01). Each of these methods is characterized by a single parameter, a virtual screening radius r_0 and a damping constant α respectively. We show theoretically and experimentally that asymptotically, for small and large d, where d is the distance between a conductor and a return conductor, the results of both methods are identical when the damping constant is identified as $\alpha \approx 1.123\,/r_0$. Moreover, we show that virtual screening behaves better in the intermediate regime $d \sim r_0$ and that computation of matrix elements is more efficient.

Introduction

In deep submicron technology feature sizes decrease, clock speeds get higher and chips get larger and more complex. Therefore, parasitic inductive effects in on-chip interconnects are playing an increasingly important role. The partial inductance concept [1] (PEEC) enables the use of inductive properties of open-ended conductor segments in circuit simulations. The PEEC method effectively replaces loop inductances by a sum of partial inductances associated with each of the conductor segments composing the loops. A partial inductance is a mathematical concept, as only closed current loops have physical meaning. However, the current induced in a segment may be interpreted as flowing in a loop formed by the segment and a return segment at infinity. The partial inductance is then proportional to the magnetic flux enclosed by this infinite loop. One characteristic of this approach is that it is important to include all partial inductances in the calculation of the inductive properties of the circuit. Partial inductances are comparatively large in magnitude and the final loop inductances are the result of cancellations of major parts of such large terms. Omitting one of them might introduce significant errors. Hence, almost by definition, PEEC matrices behave very badly under sparsification: truncating relatively small matrix elements can still give considerable errors in the overall inductive behaviour. Therefore, a number of approaches have been proposed to improve the robustness of PEEC under sparsification [2][3][4][5].

In this paper, we explore the relationship between the virtual screening approach of [5] and the damped potential method of [3]. Each of these methods is characterized by a single parameter, a virtual screening radius r_0 and a damping constant α respectively. We establish a unique relation between these parameters, on the basis of asymptotic behaviour of the respective partial inductance expressions for closely spaced filaments. This theoretical result is verified through simulations, which in addition allow us to extend the analysis to the full range of wire spacings.

Interconnects as effective filaments

The partial mutual inductance of two identical parallel conducting filaments (infinitesimal cross sections), labeled i and j respectively, of lengths l much larger than their spacing d_{ij}, and without axial offset, is given by [6]

$$L_{ij} \approx \frac{\mu_0}{2\pi}\, l \left\{ \ln\left(\frac{2l}{d_{ij}}\right) - 1 \right\} . \qquad (1)$$

When conductors a and b have finite cross sections, they may be envisaged as bundles of filaments, labeled $\{i_a\}$ and $\{j_b\}$ respectively. Accordingly, their partial mutual inductance is obtained by averaging over all filament pairs $\{i_a j_b\}$, denoted by $\langle \ldots \rangle$, as

$$L_{ab} = \langle L_{i_a j_b} \rangle . \qquad (2)$$

For l much larger than any distance $d_{i_a j_b}$ between filaments, this averaging procedure leads to

$$L_{ab} \approx \frac{\mu_0}{2\pi}\, l \left\{ \ln\left(\frac{2l}{\mathcal{D}_{ab}}\right) - 1 \right\} , \qquad (3)$$

where the *geometric mean distance* $\mathcal{D}_{ab}$, related to the cross sections of the conductors, is defined by [6] $\ln(\mathcal{D}_{ab}) = \langle \ln(d_{i_a j_b}) \rangle$. The same procedure applies to the partial *self* inductance of a conductor, which yields

$$L_{pp} \approx \frac{\mu_0}{2\pi}\, l \left\{ \ln\left(\frac{2l}{\mathcal{R}_p}\right) - 1 \right\} \qquad (p = a, b) , \qquad (4)$$

where the *geometric mean radius* $\mathcal{R}_p$ is defined by $\ln(\mathcal{R}_p) = \left\langle \ln(d_{i_p j_p}) \right\rangle$, i.e. $\mathcal{R}_p \equiv \mathcal{D}_{pp}$. A key observation is, that for long parallel wires with uniform cross sections, partial self as well as mutual inductances can be written as partial mutual inductances of effective *filaments*.

As a link to the practical use of these equations for the model system studied in this paper, we summarize some well known properties [6]. The geometric mean radius of a single cylindrical conductor, with ρ the radius of its circular cross section, is $\mathcal{R} = e^{-1/4}\rho$. The geometric mean

Proceedings 6th IEEE Workshop on Signal Propagation on Interconnects, May 12-15, 2002, Castelvecchio Pascoli, Pisa, Italy, pp. 29-32

distance of a pair of cylindrical conductors is $\mathcal{D} = d$, with d the spacing of the cylinder axes. For wires with rectangular cross sections, characterized by edge lengths T and W, partial self and mutual inductances can be calculated exactly [7][1], but no simple forms for $\mathcal{R}$ and $\mathcal{D}$ are available. However, good approximations exist [6], which are particularly accurate for the class of long wires of practical interest here, which have T and W of the same order of magnitude. The geometric mean radius is[1]

$$\mathcal{R} \approx e^{-3/2}(T + W) \; . \tag{5}$$

The geometric mean distance of two such conductors is essentially equal to the spacing of their symmetry axes [1], i.e. $\mathcal{D} = d$, equivalent to the cylindrical case.

Virtual screening

Our virtual screening approach [5] amounts to introducing modified partial inductances, associated with current loops not extending to infinity, but rather to a finite distance. The inductive properties of the overall network are invariant under this operation, but, as such smaller loops enclose less magnetic flux, the individual partial inductance values are reduced. Consequently, removing a particular *modified* partial mutual inductance from the network introduces a smaller amount of uncanceled flux than would have been the case in the original formulation. A detailed derivation of the method, directly from Maxwell's equations (through a gauge transformation of the magnetic vector potential) and alternatively from an integral (network) representation, can be found in [5]. Among various possible choices for the virtual current return paths, their location on a cylinder with radius r_0 around each "aggressor" filament appears to be particularly convenient. It preserves symmetry of the partial inductance matrix (for orthogonal systems of conductors) and its effect can be accounted for by subtracting from each matrix element a term representing the partial mutual inductance between two parallel filaments, which can be calculated at low computational costs. The choice of r_0 is guided by a priori estimates of the distribution of dominant current return paths, which should be enclosed by the shell. Typically, this would imply inclusion of nearest power/ground lines. The first matrix sparsification step consists of a windowing procedure, discarding inductive couplings with segments outside of the cylindrical shell. Further sparsification will be based on the magnitudes of the remaining matrix elements.

Consider the partial mutual inductance L_{ij} of parallel identical filaments i and j of lengths l and spacing d_{ij}, as introduced previously. Virtual screening implies supplementing filament i with an auxiliary filament i' of the same length, but at a radial distance r_0 from the "aggressor" filament j. This leads with (1) to the modified partial inductance

$$L_{ij} \quad \rightarrow \quad L_{ij} - L_{ii'} \approx \frac{\mu_0}{2\pi}\, l\, \ln\left(\frac{r_0}{d_{ij}}\right) \; . \tag{6}$$

[1] This can be deduced from $L \approx \frac{\mu_0}{2\pi} l \left\{ \ln\left(\frac{2l}{T+W}\right) + \frac{1}{2} \right\}$ and (4).

This procedure may be generalized to partial mutual and self inductances of conductors with finite cross sections, by applying it to all constituting filaments. Due to the linear nature of the virtual screening procedure, this is equivalent to transforming expressions (3) and (4) respectively, which represent magnetic coupling of *effective* filaments. We note that the logarithmic term $\ln(2l)$ in (1) is not present in (6). This is indicative of screening, and partial inductances *per unit length* have become meaningful quantities. Accordingly, the virtually screened partial *self* inductance per unit length $\widetilde{L}$ of a single conductor with geometric mean radius $\mathcal{R}$ and length $l \gg \mathcal{R}$ is

$$\widetilde{L} \approx \frac{\mu_0}{2\pi}\, \ln\left(\frac{r_0}{\mathcal{R}}\right) \; . \tag{7}$$

The virtually screened partial *mutual* inductance per unit length $\widetilde{M}$ of two parallel conductors, each of length l, without axial offset and separated by a geometric mean distance $\mathcal{D} \ll l$, is

$$\widetilde{M} \approx \frac{\mu_0}{2\pi}\, \ln\left(\frac{r_0}{\mathcal{D}}\right) \; . \tag{8}$$

Damped magnetic vector potential

Beattie and Pileggi [3] introduced a damped magnetic vector potential

$$||\vec{A}(\vec{R})|| \sim \frac{e^{-\alpha R}}{R} \; , \tag{9}$$

where $R = ||\vec{R}||$ is the radial coordinate. This replaces the regular $1/R$ behaviour. This heuristic expression is reminiscent of the Debye-Hückel theory for electrostatic screening in electrolytes. The term $e^{-\alpha R}$ can be interpreted as a spatial filter, a "soft" window of width $\sim 1/\alpha$, applied to the magnetic vector potential. This suggests a link with our virtual screening approach, where effectively the magnetic vector potential generated by a conductor is truncated beyond a finite shell.

The partial mutual inductance per unit length of two infinitely long parallel filaments i and j with spacing d_{ij}, consistent with the damped magnetic vector potential (9), is [3]

$$L_{ij}^{(\alpha)} = \frac{\mu_0}{2\pi}\, K_0(\alpha d_{ij}) \; , \tag{10}$$

where $K_0(\alpha d)$ is the modified Bessel function of the second kind of order 0 [8]. Following the arguments put forward previously, we write the partial mutual inductance of conductors a and b with *finite* cross sections as an average over filament-filament interactions, i.e.

$$L_{ab}^{(\alpha)} = \left\langle L_{i_a j_b}^{(\alpha)} \right\rangle \; , \tag{11}$$

which applies to self ($a = b$) as well as mutual ($a \neq b$) inductances. Obviously, formulation in terms of a geometric mean radius $\mathcal{R}$ or a geometric mean distance $\mathcal{D}$ respectively does not apply here, as these are related to averages of logarithmic terms. However, these quantities emerge in limiting forms of the partial inductances, as will be derived next.

Asymptotic behaviour

For $\alpha d_{ij} \equiv x \ll 1$ (i.e. $d_{ij} \ll 1/\alpha$) we use the asymptotic expansion [8]

$$\lim_{x\to 0} K_0(x) = -\left\{\ln\left(\frac{x}{2}\right)+\gamma\right\} = \ln\left(\frac{\beta}{x}\right) , \qquad (12)$$

where $\gamma = 0.57721\ldots$ (Euler's constant) and $\beta \equiv 2e^{-\gamma} \approx 1.123$. The condition $d_{ij} \ll 1/\alpha$ may be met for all filaments within a single conductor of sufficiently small cross section. The associated partial self inductance per unit length is then obtained by combining (11), (10) and (12), which gives

$$L^{(\alpha)} \approx \frac{\mu_0}{2\pi}\left\langle \ln\left(\frac{\beta/\alpha}{d_{ij}}\right)\right\rangle . \qquad (13)$$

As the geometric mean radius $\mathcal{R}$ is defined through $\ln(\mathcal{R}) = \langle \ln(d_{ij})\rangle$, we may write this as

$$L^{(\alpha)} \approx \frac{\mu_0}{2\pi}\ln\left(\frac{\beta/\alpha}{\mathcal{R}}\right) . \qquad (14)$$

Analogously, the partial mutual inductance per unit length, for sufficiently close and narrow conductors, is found in terms of the geometric mean distance $\mathcal{D}$ as

$$M^{(\alpha)} \approx \frac{\mu_0}{2\pi}\ln\left(\frac{\beta/\alpha}{\mathcal{D}}\right) . \qquad (15)$$

Comparing (14) and (15) with (7) and (8) respectively, we conclude that this limiting behaviour is equivalent to virtual screening with screening radius

$$r_0 = \beta/\alpha . \qquad (16)$$

This identification implies that for infinitely long parallel conductors with cross sections and spacings much smaller than $1/\alpha$, the damped magnetic vector potential (9) preserves the physical properties of the system, by virtue of the fact that this is the case for virtual screening [5]. However, this no longer applies when spacing is increased, as will be explored numerically below.

Numerical simulations

Model system

We consider a system of identical, long, parallel wires (Figure 1), which may represent a bus structure, typically of the type studied in [3]. The rectangular cross section of each wire has edge lengths T and W, which correspond to an approximate geometric mean radius $\mathcal{R}$ given by (5). The geometric mean distance of a pair of such wires, with spacing d of their symmetry axes, is $\mathcal{D} = d$. This subsystem constitutes a transmission line, with inductance L per unit length equal to

$$\mathsf{L} = 2\,(L - M) , \qquad (17)$$

where L is the partial self inductance of a conductor and M is the partial mutual inductance of the conductor pair,

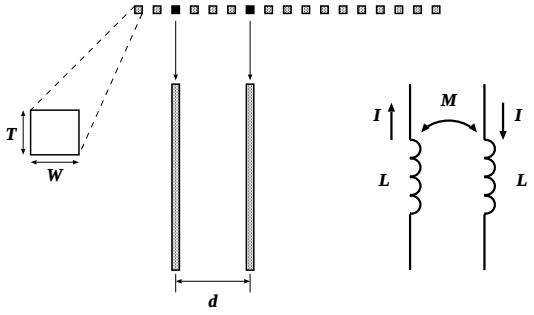

Figure 1: Parallel wires, representing a bus structure. Per unit length, the partial self inductance of a wire is L and the partial mutual inductance of a pair of wires is M.

both per unit length. We will study L as a function of the wire spacing d.

The partial inductance expressions in the virtual screening formulation (7) and (8) are

$$\widetilde{L} = \frac{\mu_0}{2\pi}\ln\left(\frac{r_0}{\mathcal{R}}\right) \quad , \quad \widetilde{M} = \frac{\mu_0}{2\pi}\ln\left(\frac{r_0}{d}\right) . \qquad (18)$$

We proved that for the damped potential the concept of a geometric mean radius $\mathcal{R}$ is valid under the condition $\mathcal{R} \ll 1/\alpha$. The geometric mean *distance*, however, does not strictly apply, as for distances d which are *not* much smaller than $1/\alpha$ the logarithmic dependence in (13) is no longer valid. However, we may still formally write the mutual inductance in terms of a distance $\mathcal{D}^{(\alpha)}$ between effective filaments through (2) and (10) as

$$M^{(\alpha)} = \frac{\mu_0}{2\pi}K_0\left(\alpha\mathcal{D}^{(\alpha)}\right) = \frac{\mu_0}{2\pi}\left\langle K_0\left(\alpha d_{i_a j_b}\right)\right\rangle . \qquad (19)$$

An upper bound for the deviation of $\mathcal{D}^{(\alpha)}$ from the regular value $\mathcal{D}$ is $(\mathcal{D}^{(\alpha)} - \mathcal{D}) \sim \mathcal{O}(\mathcal{R})$, so we may use $\mathcal{D}^{(\alpha)} \approx \mathcal{D} = d$ for $d \gg \mathcal{R}$. This is consistent with the behaviour for smaller d obtained previously, so we will use $\mathcal{D} = d$ throughout. With (16) the partial inductance expressions (14) and (19) for the damped potential are

$$L^{(\alpha)} = \frac{\mu_0}{2\pi}\ln\left(\frac{r_0}{\mathcal{R}}\right) \quad , \quad M^{(\alpha)} = \frac{\mu_0}{2\pi}K_0\left(\beta\,\frac{d}{r_0}\right) . \qquad (20)$$

Results

In the numerical computations we arbitrarily choose $r_0 = 100\,\mathcal{R}$. We first consider normalized quantities M/L, which are off-diagonal elements of the coupling matrix $\mathbf{K} = \mathbf{L}_D^{-1/2}\,\mathbf{L}\,\mathbf{L}_D^{-1/2}$, where $\mathbf{L}$ is the partial inductance matrix of the full system and $\mathbf{L}_D$ is the diagonal part of $\mathbf{L}$. Graphs of $\widetilde{M}/\widetilde{L}$ and $M^{(\alpha)}/L^{(\alpha)}$ (matrix elements of $\widetilde{\mathbf{K}}$ and $\mathbf{K}^{(\alpha)}$ respectively) as function of the normalized wire spacing d/r_0 are given in Figure 2. The fact that $\widetilde{M}/\widetilde{L} < 0$ for $d/r_0 > 1$, which looks unusual in terms of conventional partial inductances (which are positive by definition), is a mere mathematical consequence of the virtual screening procedure. Circuit simulation with $\widetilde{\mathbf{K}}$, which contains

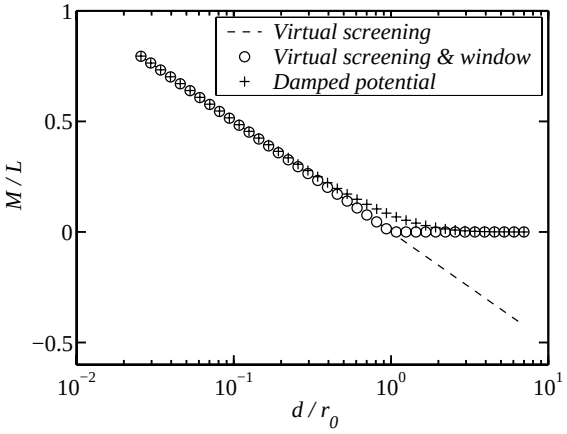

Figure 2: Normalized partial mutual inductance per unit length M/L of a conductor pair as function of the normalized wire spacing d/r_0. The screening radius is $r_0 = 100\,\mathcal{R}$. $---$ Virtual screening (18); $\bigcirc$ Windowed virtual screening; $+$ Damped potential (20).

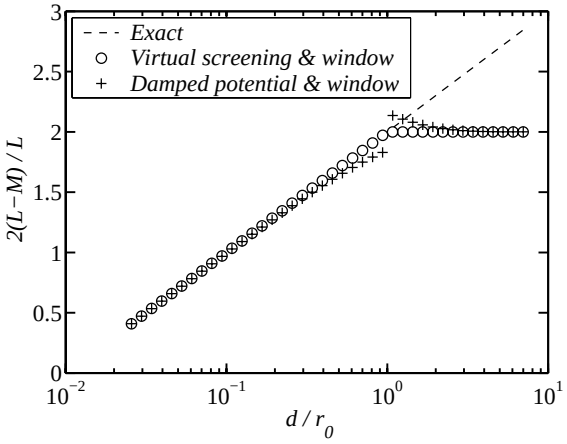

Figure 3: Normalized inductance per unit length L/L of a conductor pair as function of the normalized wire spacing d/r_0, employing a windowing procedure as described in the text. The screening radius is $r_0 = 100\,\mathcal{R}$. $\bigcirc$ Windowed virtual screening (18); $+$ Windowed damped potential (20).

these negative inductive elements, would give the same result as $\mathbf{K}$. The windowing step, however, where couplings between segments separated by a distance d larger than r_0 are discarded, implies setting these negative matrix elements in $\widetilde{\mathbf{K}}$ to zero. In fact one would not compute these terms at all, but rather remove them beforehand, based on their associated relative segment positions. Upon applying a window with size $d = r_0$ to both $\widetilde{\mathbf{K}}$ and $\mathbf{K}^{(\alpha)}$, truncated matrices with the same sparsity are obtained. However, according to [3], the discarded off-diagonal elements of $\mathbf{K}^{(\alpha)}$ should be added to the corresponding diagonal terms in order to preserve stability. We apply this procedure to the simple 2×2 matrix of the transmission-line subsystem. Figure 3 shows the effect of both procedures on its normalized inductance per unit length L/L .

Discussion

Figure 2 numerically confirms the theoretically derived equivalence, for small wire spacings $d \ll r_0 = \beta/\alpha$, of the virtual screening framework (18) and the damped potential approximation (20). Moreover, $M^{(\alpha)} \to 0$ for $d \gg r_0$, which is equivalent to the *windowed* virtual screening behaviour. As a consequence, the partial inductance per unit length L (17) of the model transmission line (Figure 1) yields unique behaviour in either limit, irrespective of the method, i.e. $\mathsf{L} = 2(L-M)$ for $d \ll r_0$ and $\mathsf{L} = 2L$ for $d \gg r_0$. However, significant differences arise in the intermediate regime of wire spacings $d \sim r_0$. These are caused by both the "smoothness" of the "exponential window" and the discontinuity necessary for preserving stability in the damped potential method. The latter is an unfortunate artefact, since it introduces an uncertainty into the chip design and analysis procedure. A small, seemingly inconsequential, layout modification might result in a rather large and unpredictable change in the waveform response in a simulation. Moreover, it may limit the allowable sparsification, because in a multi-conductor system, *all* truncated values are added to the corresponding self inductances. The latter point may be especially severe when multiple conductors carry currents in different directions, as in reality these would contribute to the loop inductance with different signs. Here it may be emphasized that virtual screening is stable without such an ad-hoc correction of diagonal matrix elements. Finally, we note that the virtual screening framework allows cheaper construction of the PEEC matrix: computation of a logarithm versus evaluation of a Bessel function for each term otherwise.

References

[1] A. E. Ruehli, "Inductance calculations in a complex integrated circuit environment," *IBM J. Res. Develop.*, vol. 16, pp. 470–481, 1972.

[2] B. Krauter and L. T. Pileggi, "Generating sparse partial inductance matrices with guaranteed stability," in *Proc. ICCAD*, 1995, pp. 45–52.

[3] M. W. Beattie and L. T. Pileggi, "Modeling magnetic coupling for on-chip interconnect," in *Proc. Design Automation Conference"*, 2001, pp. 335–340.

[4] M. Beattie, L. Alatan, and L. Pileggi, "Equipotential shells for efficient partial inductance extraction," in *Proc. International Electron Devices Meeting*, 1998, pp. 303–306.

[5] A. J. Dammers and N. P. van der Meijs, "Virtual screening: A step towards a sparse partial inductance matrix," in *Proc. IEEE/ACM International Conference on Computer Aided Design*, 1999, pp. 445–452.

[6] F. W. Grover, *Inductance Calculations: Working Formulas and Tables*, Instrument Society of America, 1945.

[7] C. Hoer and C. Love, "Exact inductance equations for rectangular conductors with applications to more complicated geometries," *J. Res. Natl. Bureau Standards*, vol. 69C, no. 2, pp. 127–137, 1965.

[8] M. Abramowitz and I.A. Stegun, *Handbook of mathematical functions*, Dover, New York, 1970.