n-Channel Symmetric Multiple-Description Lattice Vector Quantization

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Abstract

We derive analytical expressions for the central and side quantizers in an n-channel symmetric multiple-description lattice vector quantizer which, under high-resolution assumptions, minimize the expected distortion subject to entropy constraints on the side descriptions for given packet-loss probabilities. The performance of the central quantizer is lattice dependent whereas the performance of the side quantizers is lattice independent. In fact the normalized second moments of the side quantizers are given by that of an L-dimensional sphere. Furthermore, our analytical results reveal a simple way to determine the optimum number of descriptions. We verify theoretical results with numerical experiments and show that with a packet-loss probability of 5%, a gain of 9.1 dB in MSE over state-of-the-art two-description systems can be achieved when quantizing a two-dimensional unit-variance Gaussian source using a total bit budget of 15 bits/dimension and using three descriptions. With 20% packet loss, a similar experiment reveals an MSE reduction of 10.6 dB when using four descriptions.

1 Introduction

Multiple description coding (MDC) aims at creating separate descriptions individually capable of reproducing a source to a certain accuracy and when combined being able to refine each other. The classical scheme involves two descriptions, see Fig. 1. The total rate R is split between the two descriptions, i.e. $R = R_0 + R_1$, and the distortion observed at the receiver depends on which descriptions arrive. If both descriptions are received, the central distortion, d_c, is lower than if only a single description is received, d_0 or d_1.

Existing MDC schemes can roughly be divided into three categories: quantizer based, transform based and erasure codes based. Quantization based schemes include scalar quantization [1, 2], trellis coded quantization [3–5] and vector quantization [6–11]. Transform based approaches include correlating transforms [12–14] and overcomplete expansions [15–18]. Recently, schemes based on erasure codes have been introduced [19–22]. For further details on many existing MDC techniques we refer to the survey article by Goyal [23]. The present work is based on lattice vector quantization and belongs therefore to the first of the categories mentioned above.

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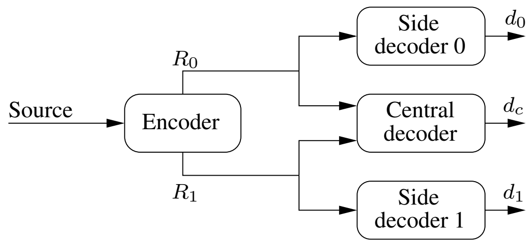

Figure 1: The traditional two-channel MDC scheme.

Practical lattice vector quantizer (LVQ) based schemes for two descriptions have been introduced in [7–11], which in the limit of infinite-dimensional source vectors approach the MDC rate-distortion bound. These schemes exploit the idea of having only one central quantizer followed by an index-assignment algorithm that maps each central quantizer reconstruction point to pairs of side quantizer reconstruction points. Given side entropies and maximum side distortions, these schemes minimize the central distortion.

In this paper we derive analytical expressions for the central and side quantizers which, under high-resolution assumptions, minimize the *expected distortion* at the receiving end of the channel subject to entropy constraints on the side descriptions for given packet-loss probabilities. The central and side quantizers we use are LVQs as presented in [9, 10].

While state-of-the-art quantizer based MDC schemes [9, 10] mainly deal with only two descriptions, we construct balanced quantizers for an *arbitrary* number of descriptions. In the presented approach the expected distortion observed at the receiving side depends only upon the number of received descriptions, hence the descriptions are mutually refinable and reception of any κ out of K descriptions yields equivalent expected distortion. We construct a scheme which, for given packet-loss probabilities and a maximum bit budget (target entropy), determines the optimal number of descriptions. It is worth noting that we present closed form expressions for the optimal quantizers, hence avoiding any iterative quantizer design procedures. Thus, our design allows for fast and easy adaptation to changes in entropy-constraints and source-channel characteristics. Furthermore, the scheme is lattice independent and can in fact be generalized to include non-lattice quantizers as well.

We verify theoretical results with numerical experiments, and show that allowing more than two descriptions offers significant performance gains. Specifically, with a packet-loss probability of 5%, a gain of 9.1 dB in MSE over state-of-the-art two-description systems can be achieved when quantizing a two-dimensional unit-variance Gaussian source using a total bit budget of 15 bits/dimension and using three descriptions. With 20% packet loss, a similar experiment reveals an MSE reduction of 10.6 dB when using four descriptions.

This paper is structured as follows. In Section 2 we briefly review specific lattice properties and introduce the concept of index-assignment. Reconstruction of the source and optimal construction of the index-assignment algorithm is presented in Section 3. In Section 4 we present a high-resolution analysis of the expected distortion and numerical evaluation follows in Section 5.

2 Preliminaries

In this work we use lattices as vector quantizers [24,25]. In the following we briefly review lattice properties and state important rate and distortion results for LVQs. We refer the reader to the cited works for further details.

2.1 Lattice properties

Let $\Lambda \in \mathbb{R}^L$ be a real lattice with Voronoi regions $V(\lambda), \lambda \in \Lambda$, given by

$$V(\lambda) \triangleq \{x \in \mathbb{R}^L : \|x-\lambda\|^2 \leq \|x-\lambda'\|^2, \forall\, \lambda' \in \Lambda\},$$

where $\|x\|^2 = \frac{1}{L}x^t x$. The dimensionless normalized second-moment of inertia $G(\Lambda)$ is given by [25]

$$G(\Lambda) \triangleq \frac{1}{\nu^{1+2/L}} \int_{V(0)} \|x\|^2 dx, \tag{1}$$

where $V(0)$ is the Voronoi region around origo and ν is the volume of a Voronoi region.

In this paper we consider one central lattice (central quantizer) Λ_c and several sublattices (side quantizers) Λ_i, where $i = 0, \dots, K-1$ and $K > 0$ is the number of descriptions. The trivial case $K = 1$ leads to a single-description system, where we would simply use one central quantizer and no side quantizers. We will consider the balanced situation, where the side entropies R_i are the same for all descriptions. Furthermore, we consider the case where the contribution d_i of each description to the total distortion is the same. Our design makes sure that the distortion observed at the receiving side, depends only on the number of descriptions received, hence reception of any κ out of K descriptions yields equivalent expected distortion. To achieve this balance, all sublattices are geometrically similar, i.e. they can be obtained from the central lattice by applying a change of scale, a rotation and possible a reflection. The sublattice index (or order) $N = [\Lambda_c : \Lambda_i], N \in \mathbb{Z}^+$, of sublattice Λ_i describes the volume of a sublattice cell relative to the volume of a central lattice cell. More specifically, the index value denotes the number of central lattice points within the Voronoi region of one sublattice point. It follows that in our case all sublattices have the same index value.

2.2 Index assignments

In the MDC scheme considered in this paper, a source vector x is quantized to the nearest reconstruction point λ_c in the central lattice Λ_c. Hereafter follows index assignments (mappings), which uniquely maps all λ_c's to vectors in each of the sublattices. This mapping is done through a labeling function α, and we denote the individual component functions of α by $\alpha_i, i = 0, \dots, K-1$. In other words, the injective map α that maps Λ_c into $\Lambda_0 \times \cdots \times \Lambda_{K-1}$, is given by

$$\begin{aligned}\alpha(\lambda_c) &= (\alpha_0(\lambda_c), \alpha_1(\lambda_c), \dots, \alpha_{K-1}(\lambda_c)) \\ &= (\lambda_0, \lambda_1, \dots, \lambda_{K-1}),\end{aligned}$$

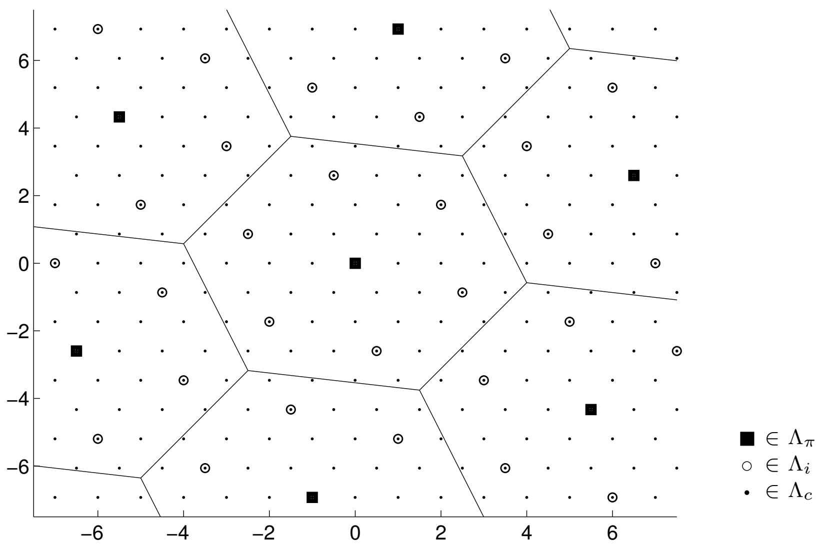

Figure 2: The A_2 lattice is used here for the central lattice (dots), a sublattice of index 7 (circles) and a sublattice of index 49 (squares). The hexagons illustrate the boundaries of the Voronoi regions V_π of Λ_π.

where $\alpha_i(\lambda_c) = \lambda_i \in \Lambda_i$. Each K-tuple $(\lambda_0, \dots, \lambda_{K-1})$ is used only once for labeling points in Λ_c in order to make sure that λ_c can be recovered unambiguously when all K descriptions are received.

Since lattices are infinite arrays of points, we construct a shift invariant labeling function, so that we only need to label a finite number of points in some predefined region, which can be shifted throughout the lattice to cover the whole space (see [9, 10, 26]). Specifically, we construct a sublattice Λ_π which has N^2 central lattice points and N sublattice points from each sublattice in each of its Voronoi regions. The Voronoi regions V_π of the sublattice Λ_π are all similar so by concentrating on labeling only central lattice points within one Voronoi region of Λ_π, the rest of the central lattice points may be labeled simply by translating this Voronoi region throughout $\mathbb{R}^L$. Other choices of Λ_π are possible, but this choice has a particular simple construction, see Fig. 2 for an example. We only label central lattice points within $V_\pi(0)$, which is the Voronoi region of Λ_π around origo. Exploiting the shift-invariance property of V_π makes it possible to write the mapping as

$$\alpha(\lambda_c + \lambda_\pi) = \alpha(\lambda_c) + \lambda_\pi, \tag{2}$$

for all $\lambda_\pi \in \Lambda_\pi$ and all $\lambda_c \in \Lambda_c$.

2.3 Rate and distortion performance of MD LVQs

Using standard high-resolution assumptions for lattice quantizers [27], the expected central distortion can be expressed as

$$d_c \approx G(\Lambda_c)\nu^{2/L}, \tag{3}$$

and it can be shown that the side distortion for the ith description is given by [26]

$$d_i \approx d_c + \frac{1}{N^2} \sum_{\lambda_c \in V_\pi(0)} \|\lambda_c - \alpha_i(\lambda_c)\|^2, \quad i = 0, \dots, K-1. \tag{4}$$

The minimum entropy R_c needed to achieve the central distortion d_c is given by

$$R_c \approx h(X) - \frac{1}{L} \log_2(\nu), \tag{5}$$

where $h(X)$ is the component-wise differential entropy of the source. The side entropies R_i are all equal and given by [26]

$$R_i = R_c - \frac{1}{L} \log_2(N). \tag{6}$$

3 Labeling function

An essential part of a multiple-description LVQ is the index assignment. The index assignment is done by a labeling function α, that maps central lattice points to K-tuples of sublattice reconstruction points. An optimal index assignment minimizes the expected distortion when $1 \leq \kappa \leq K-1$ descriptions are received. In addition the index assignment should be invertible so the central quantizer can be used when all descriptions are received.

3.1 Expected distortion

At the receiving side, $X \in \mathbb{R}^L$ is reconstructed to a quality that is determined only by the number of received descriptions. If no descriptions are received we reconstruct using the expected value, $E[X]$, and if all K descriptions are received we reconstruct using the inverse map α^{-1}, hence obtaining the quality of the central quantizer. In all other cases, we reconstruct to the average[1] of the received descriptions. There are in general several ways of receiving κ out of K descriptions. Let $\mathcal{L}$ denote an index set consisting of all possible κ combinations out of $\{0, \dots, K-1\}$ so that $|\mathcal{L}| = \binom{K}{\kappa}$. We denote an element of $\mathcal{L}$ by $l = \{l_0, \dots, l_{\kappa-1}\} \in \mathcal{L}$. Upon reception of any κ descriptions we reconstruct to $\hat{X}$ using

$$\hat{X} = \frac{1}{\kappa} \sum_{j=0}^{\kappa-1} \lambda_{l_j}. \tag{7}$$

The distortion when receiving κ out of K descriptions can be written similar to (4) by use of (7), e.g. for the case where descriptions i and j are received the norm in (4) is given by $\|\lambda_c - 0.5(\alpha_i(\lambda_c) + \alpha_j(\lambda_c))\|^2$. Assuming packet-loss probabilities are independent

[1] The average value of the received descriptions is equivalent to their centroid, since under high-resolution assumptions the pdf of X is constant within the region where elements of a K-tuple are located.

and are the same, say p, for all descriptions, we may write the expected distortion when receiving κ out of K descriptions as

$$d_a^{(K,\kappa)} \approx (1-p)^\kappa p^{K-\kappa}\left(\binom{K}{\kappa} d_c + \frac{1}{N^2}\sum_{l\in\mathcal{L}}\sum_{\lambda_c\in V_\pi(0)}\left\|\lambda_c - \frac{1}{\kappa}\sum_{j=0}^{\kappa-1}\lambda_{l_j}\right\|^2\right), \tag{8}$$

where $\lambda_{l_j} = \alpha_{l_j}(\lambda_c)$ and the two special cases $\kappa \in \{0, K\}$ are given by $d_a^{(K,0)} \approx p^K E[\|X\|^2]$ and $d_a^{(K,K)} \approx (1-p)^K d_c$. The total expected distortion d_a is obtained by summing over κ including the cases where $\kappa = 0$ and $\kappa = K$.

3.2 Optimal index assignment

In order to minimize the expected distortion, we rewrite (8) by use of the following theorem (for a proof see [26]).

Theorem 1 *For $1 \le \kappa \le K$ we have*

$$\sum_{l\in\mathcal{L}}\sum_{\lambda_c}\left\|\lambda_c - \frac{1}{\kappa}\sum_{j=0}^{\kappa-1}\lambda_{l_j}\right\|^2$$
$$= \sum_{\lambda_c}\binom{K}{\kappa}\left(\left\|\lambda_c - \frac{1}{K}\sum_{i=0}^{K-1}\lambda_i\right\|^2 + \left(\frac{K-\kappa}{K^2\kappa(K-1)}\right)\sum_{i=0}^{K-2}\sum_{j=i+1}^{K-1}\|\lambda_i-\lambda_j\|^2\right).$$

The first term on the right-hand-side in the theorem describes the distance from a central lattice point to the centroid of its associated K-tuple. The second term describes the sum of pairwise squared distances (SPSD) between elements of the K-tuples. We can show that, under a high-resolution assumption, this second term is dominant, from which we conclude that in order to minimize (8) we must use K-tuples with the smallest SPSD (for details see [26]).

The optimal labeling function minimizes (8) whenever $1 \le \kappa \le K-1$ descriptions are received. This can be posed and solved as a linear assignment problem. Let $\tilde{V} \subset \mathbb{R}^L$ be the smallest region which contains all points from e.g. Λ_j which are to be used in K-tuples also containing a specific $\lambda_i \in \Lambda_i$. It follows that in order to minimize the SPSD between a fixed λ_i and the set of points $\{\lambda_j \in \Lambda_j \cap \tilde{V}\}$ it is required that $\tilde{V}$ forms a sphere centered at λ_i. The following steps are then needed to obtain the optimal labeling function:

1. Center a sphere $\tilde{V}$ at each $\lambda_0 \in \Lambda_0 \cap V_\pi(0)$ and construct all possible K-tuples $(\lambda_0, \lambda_1, \ldots, \lambda_{K-1})$ where $\lambda_i \in \Lambda_i \cap \tilde{V}(\lambda_0)$ and $i = 1, \ldots, K-1$. This makes sure that all K-tuples have their first coordinate (λ_0) inside $V_\pi(0)$ and they are therefore shift-invariant. Make $\tilde{V}$ large enough so that at least N^2 distinct K-tuples can be found.

2. Construct cosets of each K-tuple. A coset of the K-tuple $t = (\lambda_0, \ldots, \lambda_{K-1})$ consist of the set $\{t + \lambda_\pi\}$, for all $\lambda_\pi \in \Lambda_\pi$.

3. The N^2 central lattice points in $\Lambda_c \cap V_\pi(0)$ must now be matched to distinct K-tuples. This is a standard linear assignment problem where only one member from each coset is allowed to be matched to a central lattice point in $V_\pi(0)$.

4 High-resolution analysis

Using a high-resolution analysis of (8), that is, we assume $N \to \infty$ and $N\nu \to 0$, we can show [26] that the total expected distortion $d_a = \sum d_a^{(K,\kappa)}$ can be written as

$$d_a \approx \hat{K}_1 G(\Lambda_c)\nu^{2/L} + \hat{K}_2 G(S_L)\psi^{2/L} N^{2K/L(K-1)}\nu^{2/L} + p^K E[\|X\|^2], \tag{9}$$

where $G(S_L)$ is the dimensionless normalized second moment of a sphere, $\hat{K}_1 = 1 - p^K$ and $\hat{K}_2$ is given by

$$\hat{K}_2 = \sum_{\kappa=1}^{K} \binom{K}{\kappa} p^{K-\kappa}(1-p)^{\kappa}\frac{K-\kappa}{2\kappa K}.$$

The constant ψ in (9) depends upon K and L and for the two-dimensional case $\psi \approx 2^{(K-2)/(K-1)}$ [26].

The total expected distortion can also be expressed as a function of entropies, i.e.

$$d_a = \hat{K}_1 G(\Lambda_c)2^{2(h(X)-R_c)} + \hat{K}_2\psi^{2/L} G(S_L)2^{2(h(X)-R_c)}2^{\frac{2K}{K-1}(R_c-R_s)} + p^K E[\|X\|^2],$$

where $R_s = R_i, \forall i$, and we see that the distortion due to the side-quantizers is independent of the sublattices. We remark that in [9] the authors discovered, to their surprise, that the side-quantizers normalized second moment approaches that of an L-dimensional sphere independent of which sublattice is used. The side-quantizers presented in this paper also exhibit this behavior. Furthermore, our design makes it clear why the side-quantizers are able to attain the space-filling advantage of a sphere.

4.1 Optimal ν, N and K.

We now derive expressions for the optimal ν, N and K. Using these values we are able to construct a central lattice and K sublattices. The optimal index assignment is hereafter found by using the approach outlined in Section 3. These lattices combined with their index assignment completely specify an optimal entropy-constrained multiple-description LVQ.

In order for the entropies of the side descriptions to be equal to the target entropy R_t/K, we rewrite (5) and (6) to get

$$N\nu = 2^{L(h(X)-R_t/K)} \triangleq \tau, \tag{10}$$

where τ is constant. The expected distortion may now be expressed as a function of ν,

$$d_a = \hat{K}_1 G(\Lambda_c)\nu^{2/L} + \hat{K}_2\psi^{2/L} G(S_L)\nu^{2/L}\nu^{-\frac{2K}{L(K-1)}}\tau^{-\frac{2K}{L(K-1)}} + p^K E[\|X\|^2]. \tag{11}$$

The optimal ν is easily found by differentiating (11) with respect to ν, equate to zero and solve for ν, which leads to

$$\nu = \tau\left(\frac{1}{K-1}\frac{\hat{K}_2}{\hat{K}_1}\frac{G(S_L)}{G(\Lambda_c)}\psi^{2/L}\right)^{\frac{L(K-1)}{2K}}. \tag{12}$$

The optimal N follows by use of (10)

$$N = \left((K-1)\frac{\hat{K}_1}{\hat{K}_2}\frac{G(\Lambda_c)}{G(S_L)}\frac{1}{\psi^{2/L}}\right)^{\frac{L(K-1)}{2K}}. \tag{13}$$

Eq. (13) shows that, for a fixed K, the optimal redundancy N is independent of the sublattices, the target entropy and the source.

For a fixed K the optimal ν and N is given by (12) and (13), respectively, and the optimal K can then easily be found by evaluating (9) for various values of K, and choosing the one that yields the lowest expected distortion:

$$K_{\text{opt}} = \arg\min_{K} d_a, \quad K = 1, \dots, K_{\text{max}}, \tag{14}$$

where K_{max} is a suitable chosen positive integer.

5 Numerical evaluation

In this section we compare the numerical performances of two-dimensional entropy-constrained multiple-description LVQ systems to their theoretically prescribed performances. The source signal in this example is a two-dimensional zero-mean unit-variance Gaussian signal. We use $4 \cdot 10^6$ data samples and measure the MSE when the packet-loss probability is swept from 0 to 100%. The target entropy is 15 bits/dimension for all curves, and the bits are evenly distributed over K descriptions. For example, for $K = 2$ each description uses 7.5 bits per dimension, whereas for $K = 5$ each description uses only 3 bits/dimension. Fig. 3 shows the performance of the A_2 quantizer. The practical performance of the scheme is described by the lower hull of the K-curves

It is clear that a significant performance improvement can be obtained by using more than two descriptions. In fact for $p = 5\%$ a gain of 9.1 dB is achieved by using three descriptions instead of two and for $p = 20\%$ a gain of 10.6 dB is achieved by using four descriptions instead of two. At higher packet-loss probabilities it becomes advantageous to use an even greater number of descriptions.

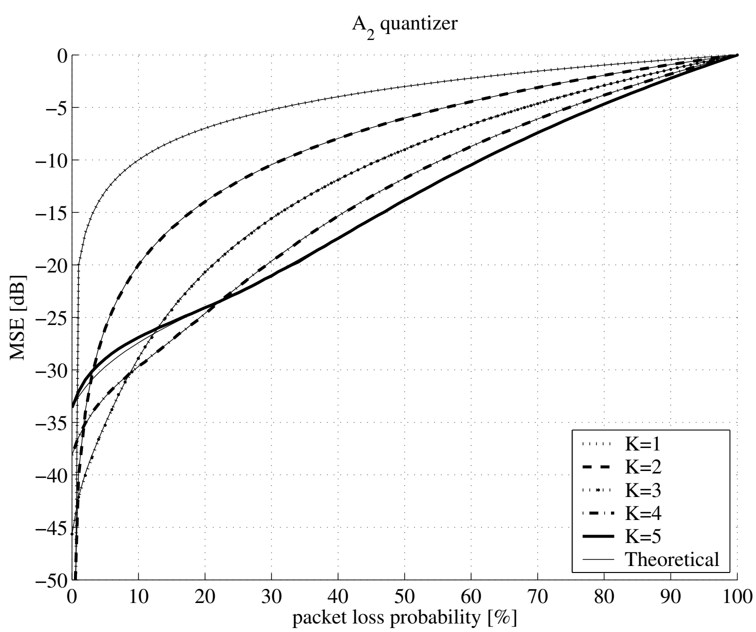

Figure 3: Expected distortion as a function of packet-loss probability for the A_2 quantizer. The target entropy is 15 bits/dimension, so each descriptions gets 15/K bits/dimension. Thick lines show numerical performance and thin solid lines show theoretical performance.

References

[1] V. A. Vaishampayan, "Design of multiple description scalar quantizers," *IEEE Trans. Inform. Th.*, vol. 39, no. 3, pp. 821 – 834, May 1993.

[2] V. A. Vaishampayan and J. Domaszewicz, "Design of entropy-constrained multiple-description scalar quantizers," *IEEE Trans. Inform. Th.*, vol. 40, no. 1, pp. 245 – 250, January 1994.

[3] V. A. Vaishampayan, J.-C. Batllo, and A. Calderbank, "On reducing granular distortion in multiple description quantization," in *Proc. IEEE Int. Symp. on Inform. Th.*, August 1998, p. 98.

[4] H. Jafarkhani and V. Tarokh, "Multiple description trellis-coded quantization," *IEEE Trans. Commun.*, vol. 47, no. 6, pp. 799 – 803, June 1999.

[5] X. Wang and M. T. Orchard, "Multiple description coding using trellis coded quantization," in *IEEE Trans. Image Proc.*, September 2001, pp. 391 – 394.

[6] M. Fleming and M. Effros, "Generalized multiple description vector quantization," in *Proc. Data Compr. Conf.*, March 1999.

[7] S. D. Servetto, V. A. Vaishampayan, and N. J. A. Sloane, "Multiple description lattice vector quantization," in *Proc. Data Compr. Conf.*, 1999, pp. 13 – 22.

[8] S. N. Diggavi, N. J. A. Sloane, and V. A. Vaishampayan, "Design of asymmetric multiple description lattice vector quantizers," in *Proc. Data Compr. Conf.*, 2000, pp. 490 – 499.

[9] V. A. Vaishampayan, N. J. A. Sloane, and S. D. Servetto, "Multiple-description vector quantization with lattice codebooks: Design and analysis," *IEEE Trans. Inform. Th.*, vol. 47, no. 5, pp. 1718 – 1734, July 2001.

[10] S. N. Diggavi, N. J. A. Sloane, and V. A. Vaishampayan, "Asymmetric multiple description lattice vector quantizers," *IEEE Trans. Inform. Th.*, vol. 48, no. 1, pp. 174 – 191, January 2002.

[11] V. K. Goyal, J. A. Kelner, and J. Kovačević, "Multiple description vector quantization with a coarse lattice," *IEEE Trans. Inform. Th.*, vol. 48, no. 3, pp. 781 – 788, 2002.

[12] M. T. Orchard, Y. Wang, V. Vaishampayan, and A. R. Reibman, "Redundancy rate-distortion analysis of multiple description coding using pairwise correlating transforms," in *Proc. IEEE Conf. on Image Proc.*, vol. 1, 1997, pp. 608 – 611.

[13] Y. Wang, M. T. Orchard, and A. R. Reibman, "Multiple description image coding for noisy channels by pairing transform coefficients," in *Proc. IEEE Workshop Multimedia Signal Processing*, 1997, pp. 419 – 424, Princeton, NJ.

[14] V. K. Goyal and J. Kovačević, "Generalized multiple descriptions coding with correlating transforms," *IEEE Trans. Inform. Th.*, vol. 47, no. 6, pp. 2199 – 2224, September 2001.

[15] R. Balan, I. Daubechies, and V. Vaishampayan, "The analysis and design of windowed fourier frame based multiple description source coding schemes," *IEEE Trans. Inform. Th.*, vol. 46, no. 7, pp. 2491 – 2536, November 2000.

[16] P. A. Chou, S. Mehrotra, and A. Wang, "Multiple description decoding of overcomplete expansions using projections onto convex sets," in *Proc. Data Compr. Conf.*, March 1999, pp. 72 – 81.

[17] J. Kovačević, P. L. Dragotti, and V. K. Goyal, "Filter bank frame expansions with erasures," *IEEE Trans. Inform. Th.*, vol. 48, no. 6, pp. 1439 – 1450, June 2002.

[18] V. Goyal, J. Kovačević, and J. Kelner, "Quantized frame expansions with erasures," *Journal of Appl. and Comput. Harmonic Analysis*, vol. 10, no. 3, pp. 203–233, May 2001.

[19] S. S. Pradhan, R. Puri, and K. Ramchandran, "n-channel symmetric multiple descriptions–part I: (n,k) source-channel erasure codes," *IEEE Trans. Inform. Th.*, vol. 50, no. 1, pp. 47 – 61, January 2004.

[20] S. Pradhan, R. Puri, and K. Ramchandran, "(n,k) source-channel erasure codes: Can parity bits also refine quality?" in *Conference on Information Sciences and Systems*, March 2001.

[21] R. Puri, S. Pradhan, and K. Ramchandran, "n-channel multiple descriptions: Theory and constructions," in *Proc. Data Compr. Conf.*, 2002.

[22] S. Pradhan, R. Puri, and K. Ramchandran, "MDS source-channel codes," in *Proc. IEEE Int. Symp. on Inform. Th.*, 2001.

[23] V. K. Goyal, "Multiple description coding: Compression meets the network," *IEEE Signal Processing Mag.*, pp. 74 – 93, September 2001.

[24] R. M. Gray, *Source Coding Theory*. Kluwer Acedemic Publishers, 1990.

[25] J. H. Conway and N. J. A. Sloane, *Sphere packings, Lattices and Groups*, 3rd ed. Springer, 1999.

[26] J. Østergaard, J. Jensen, and R. Heusdens, "n-channel entropy-constrained multiple-description lattice vector quantization," Submitted to IEEE Trans. Inform. Th.

[27] A. Gersho and R. M. Gray, *Vector Quantization and Signal Compression*. Kluwer Academic Publishers, 1991.