DISTRIBUTED DELAY AND SUM BEAMFORMER IN REGULAR NETWORKS BASED ON SYNCHRONOUS RANDOMIZED GOSSIP

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ABSTRACT
In this paper, we review and propose an improvement on an earlier proposed distributed delay and sum beamformer (DDSB). In the original DDSB, each pair of neighboring nodes updates their estimates asynchronously, which may yield a relatively slow convergence, especially when the size of network is large. When multiple pairs of neighboring nodes update simultaneously, the algorithm converges potentially faster. We propose an improved synchronous DDSB in a regular network based on synchronous randomized gossip. Moreover, we provide a comparison of the convergence rate of the asynchronous DDSB with the original and with the improved synchronous DDSB. The simulation results show the faster convergence results and show that the DDSB using the different updating schemes converges to the centralized beamformer.

Index Terms— Distributed delay and sum beamformer, synchronous randomized gossip, speech enhancement, sensor networks

1. INTRODUCTION
Speech processing applications, such as hearing aids, mobile telephony and human-machine communication systems, significantly degrade in quality and speech intelligibility in noisy environments. A common way to make these speech processing applications robust against environmental noise and interferences is to use speech enhancement algorithms. Multi-microphone technologies for speech enhancement can improve the quality and intelligibility by constructing a beamformer. However, conventional multi-microphone noise reduction algorithms require a centralized processor and consider a limited number of microphones. Modern technologies enable the use of many low cost microphones each having an individual processor in a wireless sensor network (WSN). The centralized beamformer is neither robust nor scalable for a large size WSN. Recently, several distributed algorithms are introduced to perform the multi-microphone noise reduction algorithms in a large WSN, e.g., [1] [2].

In [2], we have introduced a distributed delay and sum beamformer (DDSB) which operates in a randomly connected network using the asynchronous randomized gossip algorithm [3]. Without any topology constraint, the nodes in the network estimate the desired signal by using only local information and by performing only local processing. The nodes thus only need to perform relatively simple operations. In addition, the algorithm is robust for large sensor networks and dynamic environments. The DDSB considered in [2] is randomized and operates in an asynchronous iterative fashion. The algorithm converges asymptotically to the optimal centralized beamformer as the number of iterations increases. However, in the asynchronous DDSB [2], only one pair of neighboring nodes in a time slot can exchange their local information and update their current estimates, while all remaining nodes do not update their information. Therefore, the asynchronous DDSB converges relatively slowly in time, especially when the number of nodes is large.

An alternative approach is to have in each iteration a multiple number of updating node pairs and let them perform their updates simultaneously. However, without global information of the network this is not straightforward. In [3], in addition to an asynchronous gossip algorithm, also a randomized distributed synchronous gossip algorithm in a bounded degree graph and in an unbounded regular graph were considered. Since in the synchronous fashion multiple neighboring node pairs can estimate the signal statistics in parallel, the synchronous averaging algorithm in both graphs can potentially increase the convergence rate. In this paper, we will constrain ourselves to discuss only parallel updating in an unbounded regular graph, due to limited space. Based on the distributed synchronous averaging algorithm presented in [3], we present an improved synchronous randomized gossiping algorithm with increased probability that neighboring node pairs communicate. We prove that the proposed algorithm converges faster than the existing synchronous gossip algorithm presented in [3]. We will in addition provide a comparison of the convergence rate of the asynchronous communication algorithm with the existing and proposed synchronous communication algorithm in a regular graph. Moreover, we combine the different communication algorithms with the in [2] presented DDSB and test the convergence rate of the DDSB using the different communication schemes in a simulated scenario. The results show that the simultaneous updating significantly improves the convergence speed of the DDSB when there is a sufficient amount of nodes in the regular network. However, for a regular network with a small number of nodes it can be shown that the asynchronous randomized gossip algorithm converges faster than the synchronous communication schemes.

2. PROBLEM STATEMENT AND NOTATION
In this work we consider a randomly connected WSN setup consisting of \( N \) microphones. Each microphone \( i \) computes the noisy speech discrete Fourier transform (DFT) coefficients \( Y_i(k, m) \) on a frame-by-frame basis, where \( k \) and \( m \) denote the frequency bin and time-frame index, respectively. The DFT coefficients \( Y_i(k, m) \) for all microphones \( i \in 1, \ldots, N \) are stacked in an \( N \)-dimensional vector \( \mathbf{Y}(k, m) \). Because we assume the DFT coefficients to be independent across time and frequency, we neglect in the sequel the indices \( k \) and \( m \) for notational convenience. We assume a wireless

\[ \text{This research is partly supported by the Dutch Technology Foundation STW.} \]
In this section, in order to guide the reader, we briefly review the DDBS, while a more detailed description is given in [2]. The DDBS is a distributed beamforming algorithm that is described in Sec. 4.1 as synchronous DDSB. In this section, we give a brief overview of the distributed synchronous averaging algorithm in a regular network first, and then we present an improvement of this synchronous algorithm in the regular graph to increase the convergence rate.

4. SYNCHRONOUS COMMUNICATION

Unlike the asynchronous DDBS [2] where only one pair of nodes communicates at iteration \( t \), we consider here the case where the updating happens in a synchronous fashion, i.e., multiple node pairs communicate in the same iteration and these simultaneously active node pairs are constrained to be disjoint. The distributed synchronous averaging algorithm in [3] is considered to obtain multiple communicating node pairs in a distributed way for an unbounded degree regular graph and a bounded degree graph, respectively. Based on the synchronous gossip processing, we refer to this DDBS as synchronous DDBS. In this section, we give a brief overview of the distributed synchronous averaging algorithm in a regular network first, and then we present an improvement of this synchronous algorithm in the regular graph to increase the convergence rate.

4.1. Synchronous averaging algorithm

In a connected network, a \( d \)-regular graph is considered where each node has exactly \( d \) neighbors. At iteration \( t \), each node in the synchronous averaging algorithm is active with probability \( 1/d \) independently. When node \( i \) is active, it randomly selects one neighboring inactive node \( j \) with probability \( 1/d \) and ignores all requests of its neighboring active nodes. The inactive node \( j \) also ignores all requests of active nodes if contacted by more than one active node. If node \( i \) contacts node \( j \) but no other nodes contact node \( j \), then nodes \( i \) and \( j \) always average their current estimates.

4.2. Improved synchronous averaging algorithm

A strong limitation of the synchronous communication scheme from [3] described in Sec. 4.1 is that an inactive node \( j \) fails to have contact with any node when more than one node, say \( g \) nodes, contact node \( j \) during the same iteration. In this subsection, we propose

\[
Y = X + V, \tag{1}
\]

where the \( N \)-dimensional vectors \( X \) and \( V \) indicate the vector of speech and noise DFT coefficients, respectively, for microphones \( i = 1 \) up to \( i = N \). In this paper, we assume that the speech and noise are uncorrelated, and that a single desired speech source exists in the network. Vector \( X \) can then be written as \( X = dS \), where \( S \) is a desired speech source DFT coefficient, and \( d \) is the acoustic transfer function from the source \( S \) to all microphones. The acoustic transfer function \( d \) is determined by gain and delay values as \( d = [a_1 e^{-j\omega_k \tau_1}, \ldots, a_N e^{-j\omega_k \tau_N}]^T \), where \( a_i \) is the damping coefficient, \( \tau_i \) denotes the delay in number of samples and the superscript \( T \) denotes transpose of a vector or a matrix.

In order to estimate the clean speech DFT coefficient \( S \) at each microphone, we use a weighted linear estimator \( \hat{z} = w^H Y \) for each node \( i \) with \( w \) a filter coefficient vector and \( (\cdot)^H \) denoting Hermitian transposition. One often used spatial filter is the so-called minimum variance distortionless response (MVDR) beamformer, that is

\[
w = R_{VV}^{-1} d \quad \text{and} \quad y = w^H Y, \tag{2}
\]

where \( R_{VV} \) is defined as \( R_{VV} = E[VV^H] \) with \( E[\cdot] \) denoting the statistical expectation operator. If we assume that \( V_i \), \( V \) are zero mean, spatially uncorrelated with power spectral density (PSD) \( \sigma_{V_i}^2 \), \( R_{VV} = \text{diag} \{ \sigma_{V_1}^2, \ldots, \sigma_{V_N}^2 \} \), which allows us to write the beamformer output \( \hat{z} \) as

\[
z = \frac{\sum_{i=1}^{N} a_i \sigma_{V_i}^2 e^{j\omega_k \tau_i} Y_i}{\sum_{i=1}^{N} a_i^2 \sigma_{V_i}^2}. \tag{3}
\]

This beamformer is more general than the generally used delay and sum beamformer, as also presented in [4], as it allows for different noise PSDs per microphone. The above assumption on the noise field is reasonable for a diffuse and uncorrelated noise field and/or when the distance between microphones is sufficiently large. The optimal solution in Eq. (3) can be obtained by using a centralized beamformer. However, when the size of the WSN grows, the centralized beamformer will neither be robust nor scalable. A reasonable solution is to develop distributed speech enhancement algorithms as proposed in [2].

3. DISTRIBUTED DELAY AND SUM BEAMFORMER

In this section, in order to guide the reader, we briefly review the DDSB, while a more detailed description is given in [2]. The DDSB is a distributed beamformer that obtains the same optimal solution as Eq. (3) by using only local information and local computation. In the WSN, we assume that each node \( i \) for a given time frame has two initial values, that are, \( Y_i(0) = a_i e^{j\omega_k \tau_i} Y_i \) and \( d_i(0) = a_i^2 \sigma_{V_i}^{-2} \), where realizations of \( Y_i \) are obtained by the microphone at node \( i \), and \( d_i = a_i e^{j\omega_k \tau_i} \) can be estimated and updated using [5], and the noise PSD \( \sigma_{V_i}^2 \) can be estimated using e.g., [6]. In this paper we assume \( d_i \) is known and estimate \( \sigma_{V_i}^2 \) using an ideal voice activity detector to focus on the distributed beamformer algorithm. Let \( \tilde{Y}_i(0) \) be a stacked \( N \) dimensional vector defined as \( \tilde{Y}_i(0) = [Y_i(0), \ldots, Y_N(0)]^T \), similarly, all \( \tilde{d}_i(0) \) are stacked in an \( N \) dimensional vector \( \tilde{d}(0) \). Eq. (3) can then be obtained as

\[ Z = \tilde{Y}_{ave}/\tilde{d}_{ave}; \tag{4}\]

where \( \tilde{Y}_{ave} = \frac{1}{N} \sum_{i=1}^{N} Y_i(0) \) and \( \tilde{d}_{ave} = \frac{1}{N} \sum_{i=1}^{N} d_i(0) \) with 1 denoting the vector of all ones. The DDSB aims to find the average value \( \tilde{Y}_{ave} \) and \( \tilde{d}_{ave} \) in a distributed way. In the asynchronous DDSB, each node runs an independent Poisson clock at iteration \( t \), where the \( i \)-th node is active with probability \( \frac{1}{d} \). When node \( i \)’s clock ticks, it randomly selects and communicates with one neighboring node \( j \) with probability \( p_{ij} \). All probabilities \( p_{ij} \) can be put in an \( N \times N \) probability matrix \( p \), with \( p_{ij} > 0 \) if there is a communication link between node \( i \) and node \( j \), otherwise \( p_{ij} = 0 \). At each iteration, a node \( i \) and a node \( j \) exchange and average their current estimates. Except these two communicating nodes, other nodes in the network keep the same estimates as during the last iteration \( t - 1 \). A general vector form of the DDSB at iteration \( t \) is given by

\[ \tilde{Y}(t) = \tilde{U}(t) \tilde{Y}(t - 1), \]

\[ \tilde{d}(t) = \tilde{U}(t) \tilde{d}(t - 1), \]

\[ \tilde{Z}_i(t) = \tilde{Y}_i(t) / \tilde{d}_i(t), \]

where \( \tilde{Z}_i(t) \) is the DDSB output of node \( i \), and \( \tilde{U}(t) \) is an \( N \times N \) dimensional update matrix, which is independent across time. For two communicating nodes \( i \) and \( j \) at iteration \( t \), the update matrix is

\[ \tilde{U}(t) = I - \frac{(e_i - e_j)(e_i - e_j)^T}{2}, \]

where \( e_i = [0, \ldots, 0, 1, 0, \ldots, 0]^T \) is an \( N \)-dimensional vector with the \( i \)-th component equal to 1.
an improved communication scheme for the $d$-regular graph, which can be used for synchronous DDSB and which improves the convergence rate. The idea of the improved communication scheme is that the inactive node $j$ randomly selects one of the $g$ requesting nodes with probability $1/g$ if contacted by $g$ nodes, and we refer to this algorithm as the improved synchronous DDSB.

At iteration $t$, each node in the improved synchronous DDSB becomes active independently with probability $1/2$. An active node $i$ randomly connects one neighboring inactive node $j$ with probability $1/d$, and ignores the nodes that contact it. The inactive node $j$ then randomly connects to node $i$ with probability $1/g$ if contacted by $g$ active nodes. After that, nodes $i$ and $j$ update their estimates.

5. CONVERGENCE ANALYSIS

All nodes' local information $\hat{Y}(t)$ and $\hat{d}(t)$ are guaranteed to converge to the average value $\bar{Y}_{\text{ave}}, \bar{d}_{\text{ave}}$ as long as the update matrix $U(t)$ is a doubly stochastic matrix and the network is connected [3]. Since $\hat{Y}(t)$ and $\hat{d}(t)$ converge, their ratio $\bar{Z}(t)$ converges to the optimal output (3) as long as $\bar{d}_{\text{ave}} \neq 0$.

In order to analyze the convergence rate of the DDSB, the $\epsilon$-averaging time $T_{\text{ave}}(\epsilon, P)$ of the algorithm can in analogy with [3] be defined as the first iteration where the convergence error is smaller than a desired error $\epsilon$ with high probability $1 - \epsilon$. That is,

$$
\sup_{\Psi(t)} \inf_{t=0} \epsilon \leq \{P(C \geq \epsilon) \leq \epsilon\},
$$

with the convergence error $C = \frac{\|Y(t)-Y_{\text{ave}}\|}{\|Y(0)\|}$. The averaging time $T_{\text{ave}}(\epsilon, P)$ can be shown to be bounded by the second largest eigenvalue of expected value of the update matrix $E[U]$. That is [3],

$$
0.5 \log \epsilon^{-1} \leq T_{\text{ave}}(\epsilon, E[U]) \leq \frac{3 \log \epsilon^{-1}}{\log \lambda_2(E[U])},
$$

which is 

$$
0.5 \log \epsilon^{-1} \leq T_{\text{ave}}(\epsilon, E[U]) \leq \frac{3 \log \epsilon^{-1}}{\log \lambda_2(E[U])}.
$$

Eq. (10) shows that the averaging time of the DDSB depends on the eigenvalue $\lambda_2(E[U])$. The smaller the magnitude of this eigenvalue, the faster the convergence will become. The convergence rate of the DDSB can be analyzed for the different communication schemes using the corresponding eigenvalue $\lambda_2(E[U])$.

In a $d$-regular network, we define the probability matrix $p$ as $p_{ij} = 1/d$ if nodes $i$ and $j$ are neighbors, otherwise $p_{ij} = 0$, and we assume that this matrix $p$ is the probability matrix of the asynchronous gossip as well. The expected value of the asynchronous gossip is [3] $E_A[U] = (1 - 1/N)I + p/N$, and the expected value of the synchronous gossip $E_S[U]$ can be shown to be [3]

$$
E_S[U] = (1 - \bar{d})I + \bar{d}p,
$$

where $\bar{d} = \frac{1}{d}(1 - \frac{1}{d})d^{-1}$ and $\bar{d}_i$ is the probability that a node pair $(i, j)$ is selected to average. In the improved synchronous DDSB, we compute the probability that node $i$ connects to node $j$, as explained in Sec. 4.2, that is, node $i$ is inactive with probability $1/2$; the probability that node $i$ is active, and is one of the $g$ nodes contacting $j$, and finally selected by $j$ to connect is $(\frac{1}{d^2})^\frac{1}{g}$ with $g \in \{1, \cdots, d\}$; the $d - g$ remaining nodes do not connect with probability $(1 - \frac{1}{d})^{d-g}$; after which nodes $i$ and $j$ average their values. Notice that there are $r = (\frac{d}{g}-1)$ number of combinations of selecting $g - 1$ active nodes out of $d - 1$ neighbors. Altogether, the expected value of the improved synchronous gossip $E_I[U]$ in a $d$-regular graph is then given as

$$
E_I[U] = (1 - \bar{d}_I)I + \bar{d}_I p
$$

where $\bar{d}_I = \frac{1}{d} \sum_{g=1}^{d} (\frac{1}{d^2})^\frac{1}{g}(1 - \frac{1}{d})^{d-g}$.

Since the convergence rate is determined by the magnitude of $\lambda_2(E[I])$, the convergence rate comparison between the improved synchronous DDSB and the synchronous DDSB is given by

$$
\lambda_2(E_I[U]) - \lambda_2(E_S[U]) = (\bar{d} - \bar{d}_I) (1 - \lambda_2(p)).
$$

Since $-1 \leq \lambda_2(p) < 1$, we have $\lambda_2(E_I[U]) - \lambda_2(E_S[U]) \leq 0$ for all $N > 2$ as $d \leq \bar{d}_I$. It is obvious that the improved synchronous DDSB converges faster than the synchronous DDSB with high probability.

To compare the convergence rate of the synchronous DDSB and the asynchronous DDSB, their eigenvalues can be compared as

$$
\lambda_2(E_S[U]) - \lambda_2(E_A[U]) = \frac{1 - N\bar{d}}{N} (1 - \lambda_2(p)),
$$

with $-1 \leq \lambda_2(p) < 1$ and consequently, $\frac{1 - \lambda_2(p)}{N} > 0$. Assuming that $N > 2$, the number of neighbors is given by the range $2 \leq d \leq N - 1$. Since $\bar{d}$ is a monotonically decreasing function, we have

$$
\frac{1}{4} \left(1 - \frac{1}{2(N-1)}\right)^{N-2} \leq \bar{d} \leq \frac{3}{16}.
$$

From the bound given in Eq. (15), in combination with Eq. (14) and the fact that the upper bound of $1 - N\bar{d}$ is a monotonically decreasing function, it then follows that $\lambda_2(E_S[U]) - \lambda_2(E_A[U]) > 0$ for $N \leq 5$ and $\lambda_2(E_S[U]) - \lambda_2(E_A[U]) < 0$ for $N \geq 7$, which implies that the synchronous DDSB converges slower than the asynchronous DDSB with high probability if $N \leq 5$ and with high probability the synchronous DDSB converges faster than the asynchronous DDSB if $N \geq 7$.

Using the second largest eigenvalues of the improved synchronous DDSB $\lambda_2(E_I[U])$ and the asynchronous DDSB algorithm $\lambda_2(E_A[U])$, their convergence rate comparison can be given as $\frac{1 - N\bar{d}}{N} (1 - \lambda_2(p))$. Since $2 \leq d \leq N - 1$ and $\bar{d}_I$ is monotonically decreasing as a function of $N$, $\bar{d}_I$ can be bounded as

$$
\frac{1}{4} \sum_{g=1}^{N-1} \left(\frac{1}{2(N-1)}\right)^{g-1} \frac{1}{g} \left(1 - \frac{1}{2(N-1)}\right)^{(N-1)-g} \leq \bar{d}_I \leq \frac{7}{42}.
$$

Given this bound of $\bar{d}_I$, the upper bound of $1 - N\bar{d}_I$ is a monotonically decreasing function. Then we have that $\lambda_2(E_I[U]) > \lambda_2(E_A[U])$ for $N \leq 4$, and $\lambda_2(E_I[U]) < \lambda_2(E_A[U])$ for $N \geq 5$. This means that with high probability, the improved synchronous DDSB converges faster than the asynchronous DDSB if $N \geq 5$, while the improved synchronous DDSB converges slower than the asynchronous DDSB if $N \leq 4$.

6. SIMULATIONS

In this section, we illustrate the performance of all presented algorithms via a simulated WSN. We first compare the convergence rate of the asynchronous averaging algorithm to the synchronous communication schemes in different size regular networks using synthetic data. After that, we show the performance of the synchronous DDSB and the improved synchronous DDSB on speech data.

To demonstrate the convergence analysis of the averaging algorithms, we simulate the four simple networks where 4, 5, 6 and 7 nodes are fully connected. For each graph, we consider that node
in the network has the initial value $V_i$, and the $V_i$, ∀$i$ are independent and identically distributed zero-mean Gaussian sensor noise variables. Fig. 1(a) shows that the asynchronous scheme converges faster than the improved synchronous averaging algorithm. Fig. 1(b), Fig. 1(c) and Fig. 1(d) show that the asynchronous averaging algorithm converges slower than the improved synchronous averaging algorithm when $N ≥ 5$. In Fig. 1(a) and Fig. 1(b), we show that the asynchronous averaging algorithm converges faster than the synchronous averaging algorithm if $N ≤ 5$. The asynchronous averaging algorithm converges slower than the synchronous averaging algorithm in Fig. 1(d). It is obvious that these simulation results corroborate the convergence analysis given in Sec. 5.

$$\text{MSE}_i = \frac{1}{MK} \sum_{m=1}^{M} \sum_{k=1}^{K} \left| \hat{Z}_i(k, m) - S(k, m) \right|^2,$$  \hspace{1cm} (17)

where $K$ and $M$ denote the number of frequency bins and time frames, respectively. $Z_i(k, n)$ is a DFT coefficient of the beamformer output.

Fig. 2 shows the MSE between all presented DDSB outputs of node 1 and the clean speech, compared to the MSE between the optimal centralized delay and sum beamformer (CDSB) output and the desired speech. We observe that, the DDSB using the different communication schemes reaches the same performance as the CDSB with enough iterations. In Fig. 2(a), we show a simulation result of node 1 in a fully connected network. The simulation result of node 1 in a ring connected network is shown in Fig. 2(b). Without surprise, the improved synchronous DDSB converges faster than the original synchronous DDSB and the asynchronous DDSB in both figures, since there are enough nodes in the regular graphs.

7. CONCLUSIONS

In this paper, we proposed an improved synchronous DDSB for regular networks with an increased amount of communicating neighboring nodes. The DDSB algorithm using the different communication schemes converges asymptotically to the centralized beamformer. We compared the convergence rate of the improved synchronous DDSB with the synchronous DDSB and the asynchronous DDSB in an unbounded regular graph. The simulation results show the effectiveness of the DDSB and show that the simultaneous updating increases the convergence rate of the DDSB when there is a sufficient amount of nodes in the regular network.

8. REFERENCES


