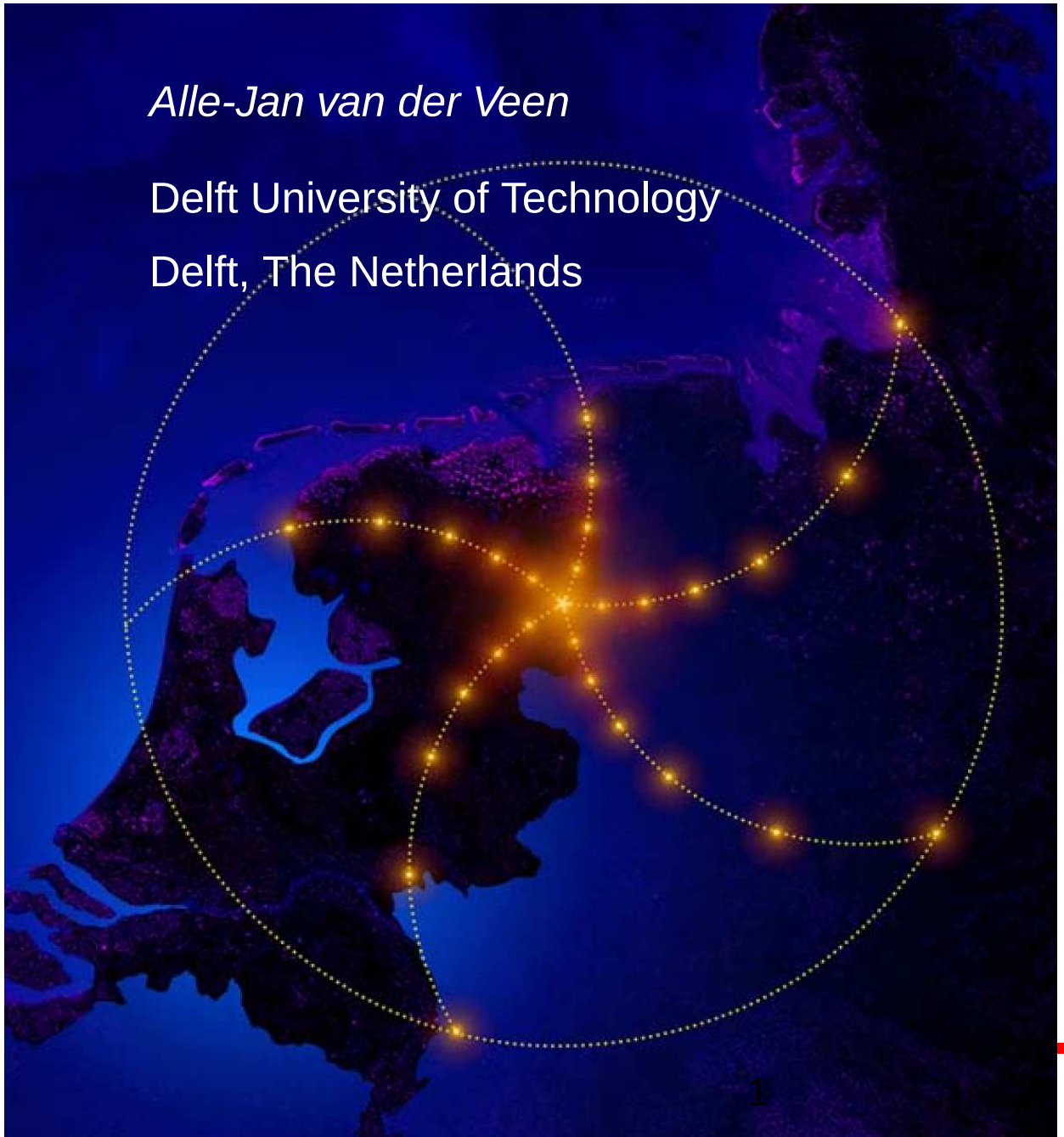
Self-calibration for the LOFAR radio astronomical array



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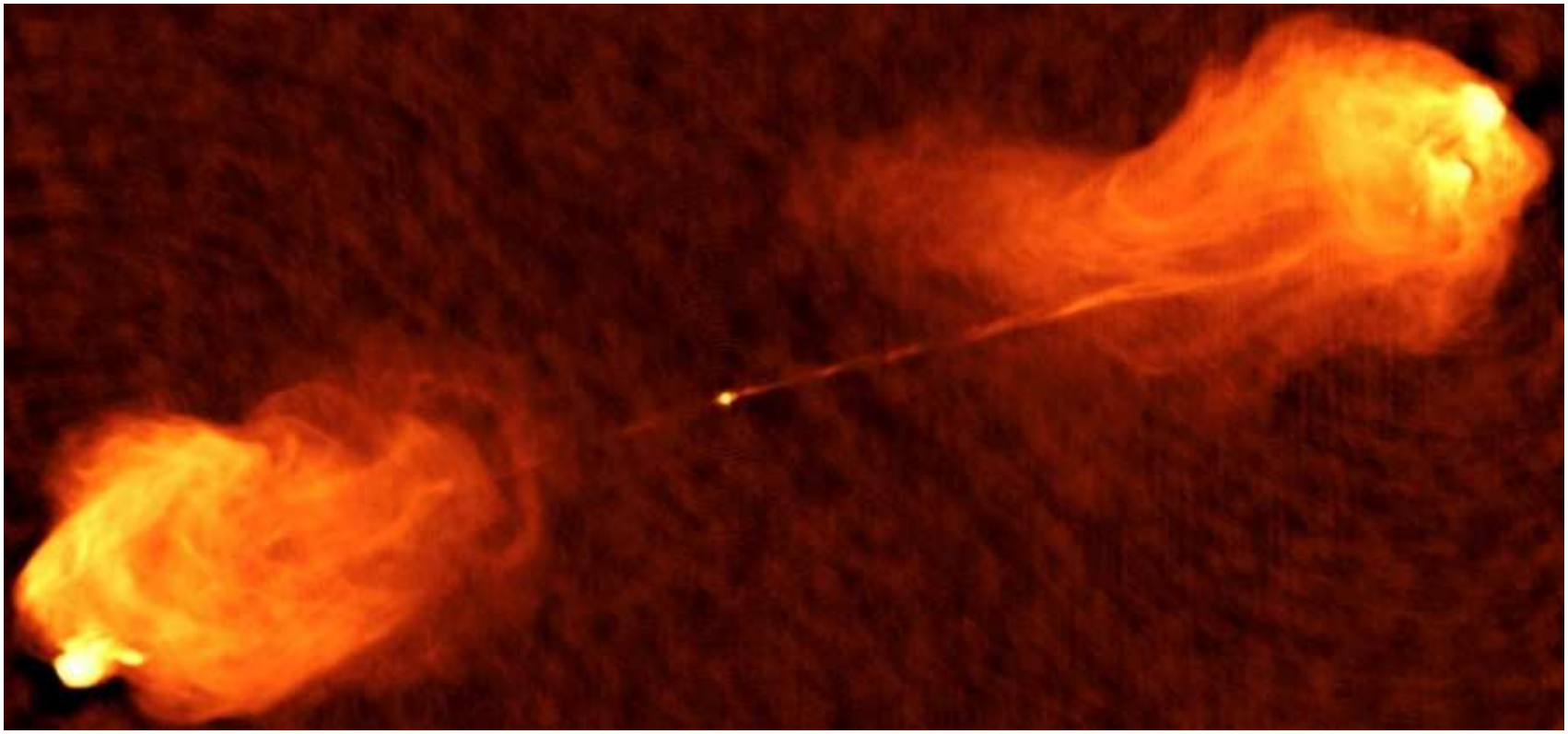
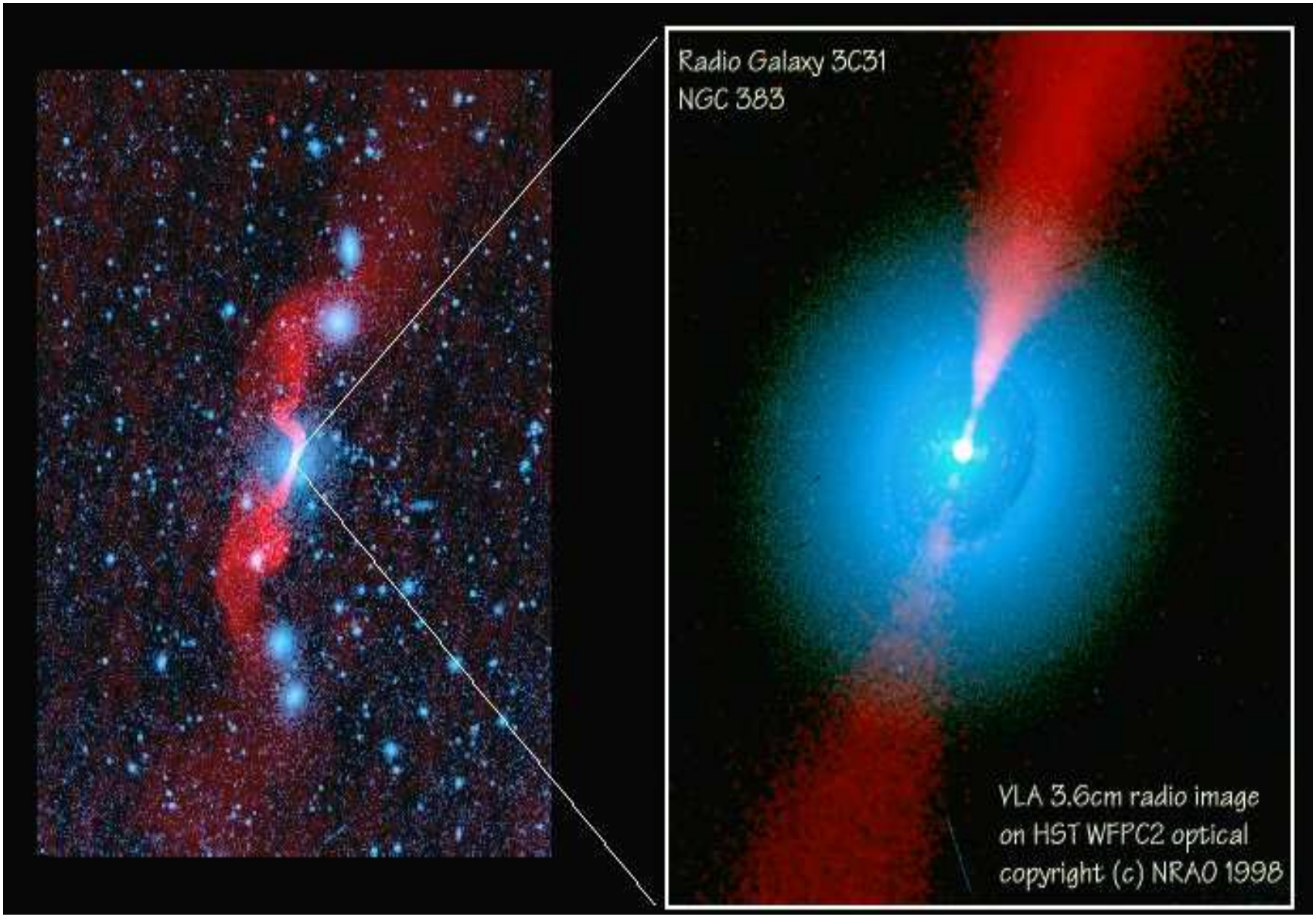
Introduction—the science

Cygnus A (a quasar)



Introduction—the science

3C31 (a radio galaxy; optical and radio overlay image)



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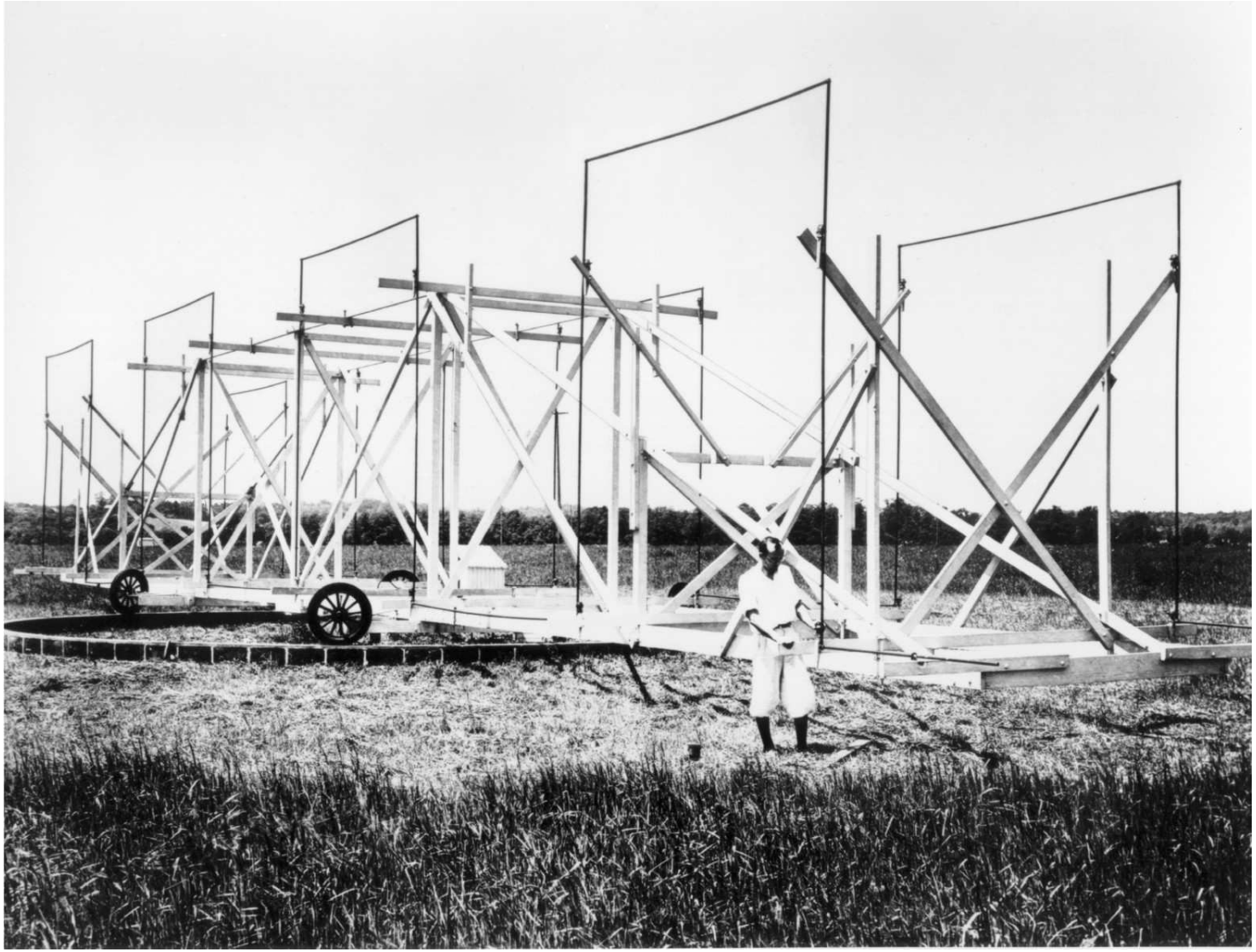
Introduction—the instruments

Karl Jansky, 1928



Introduction—the instruments

Effelsberg, Germany, 1972, 100m

Arecibo, Puerto Rico, 1960, 305m

Introduction—the instruments

Westerbork, Netherlands

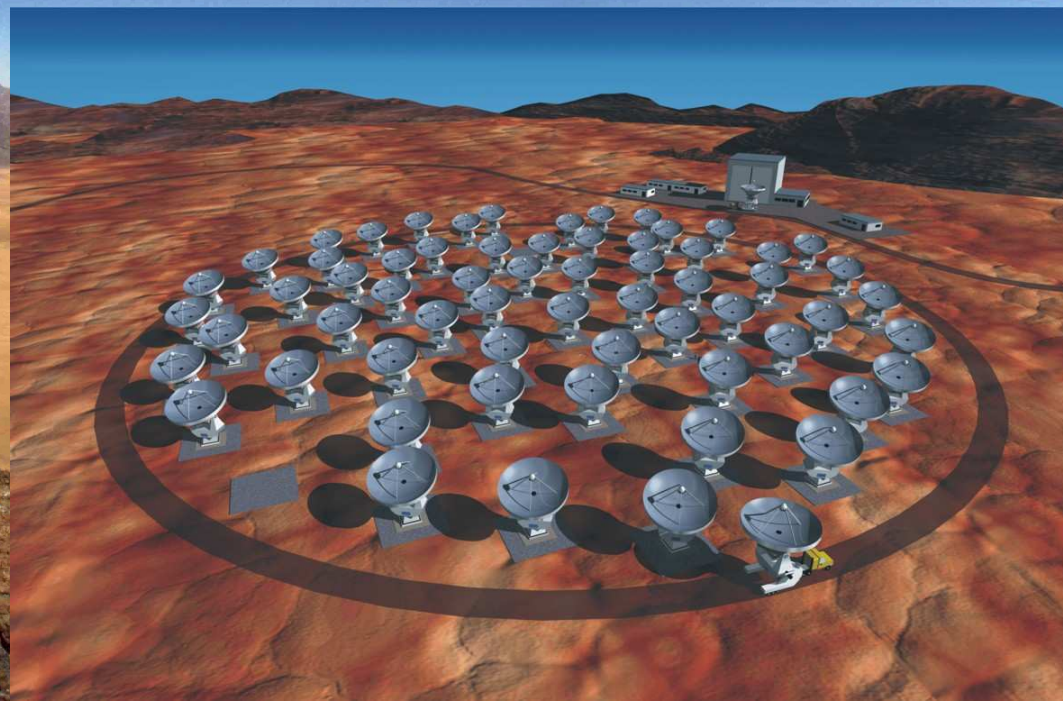
1970, 14 dishes, 3 km

Very Large Array (VLA), New Mexico

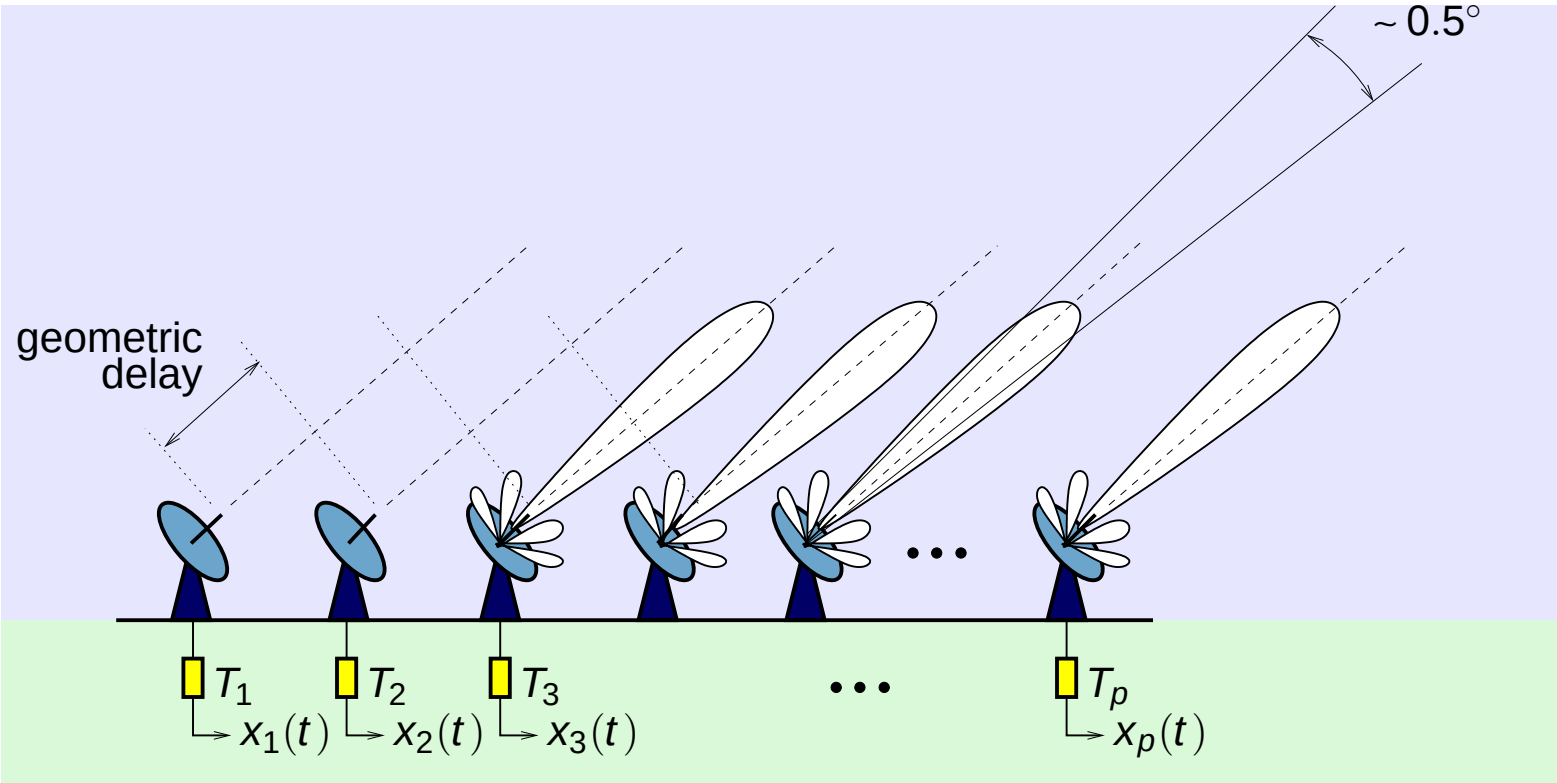
1980, 27 dishes, 36 km

Introduction—the instruments

Atacama Large Millimeter Array, Chile, 64 dishes, est. 2011



The correlation process

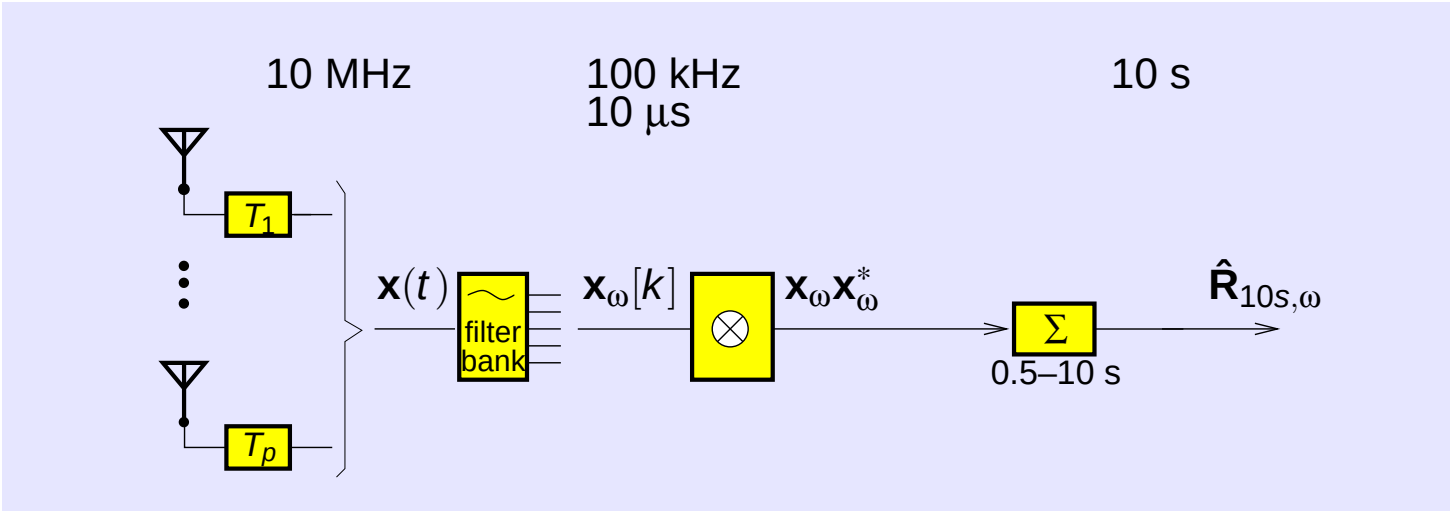


- Signals are stacked in vectors: $\mathbf{x}(t) = \begin{bmatrix} x_1(t) \\ \vdots \\ x_p(t) \end{bmatrix}$

- The location of antenna i at time t is $\mathbf{r}_i(t)$.

  The vector $\mathbf{r}_i(t) - \mathbf{r}_j(t)$ between antennas i and j is a *baseline*.

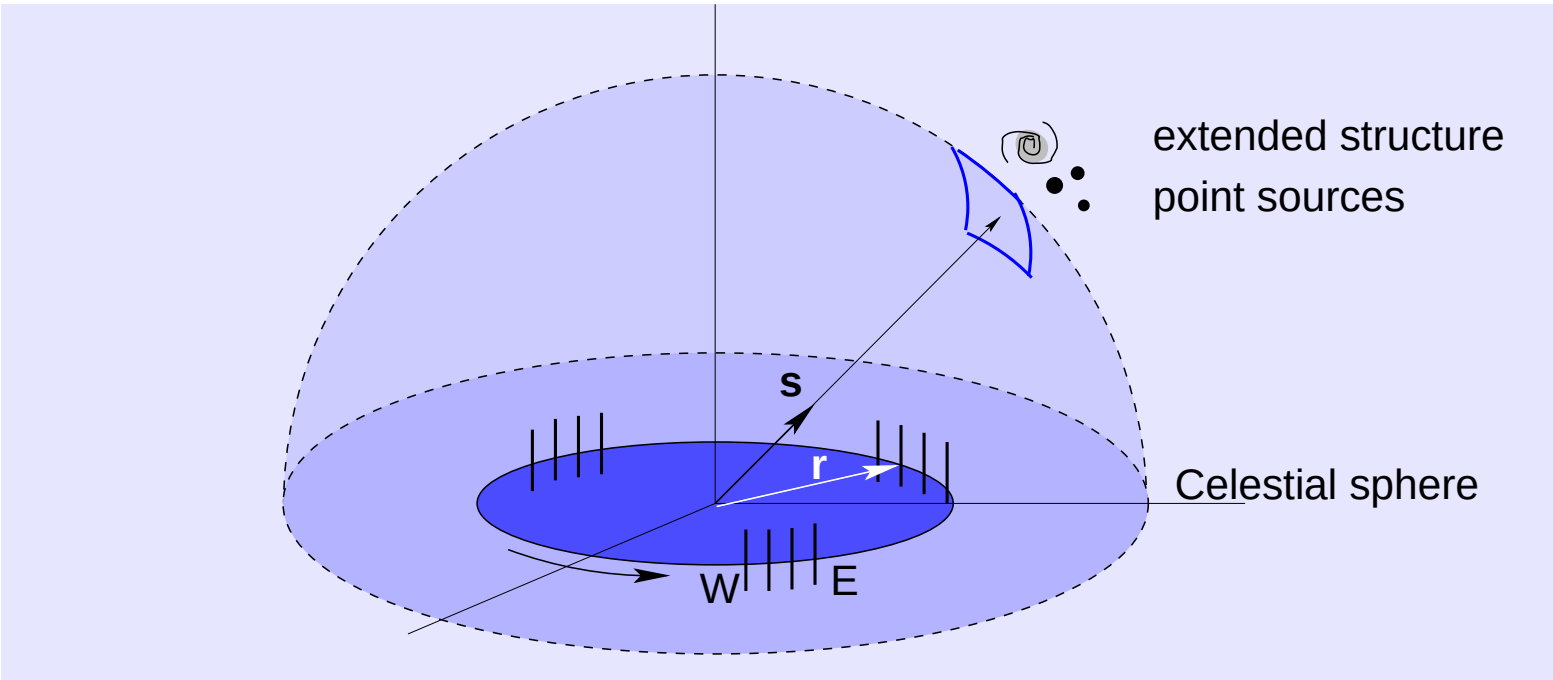
The correlation process



$$\mathbf{R}_{k,\omega} = \mathrm{E}\{\mathbf{x}_\omega(t_k)\mathbf{x}_\omega(t_k)^{\mathrm{H}}\} \simeq \frac{1}{N}\sum_{nT=0}^{10s}\mathbf{x}_\omega(t_k+nT)\mathbf{x}_\omega(t_k+nT)^{\mathrm{H}}$$

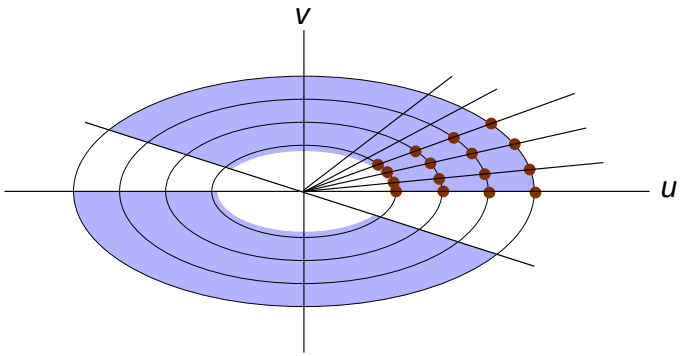
- Entry (i,j) of $\mathbf{R}_{k,\omega}$ corresponds to a *visibility* $V[\mathbf{r}_i(t_k),\mathbf{r}_j(t_k)]$ at frequency ω.

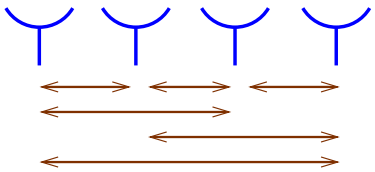
Astronomical imaging



- We can measure correlations only for a discrete set of baselines $\mathbf{r}_i(t_k) - \mathbf{r}_j(t_k)$:

  (they are represented in normalized (u, v)-*coordinates*)





Imaging—classical beamforming

Simplified measurement equation

$$V_{ijk} \equiv V[\mathbf{r}_i(t_k), \mathbf{r}_j(t_k)] \quad = \quad \sum_{\ell=1}^{d} I(\mathbf{s}_\ell)\, e^{-j\,\mathbf{s}_\ell^T(\mathbf{r}_i - \mathbf{r}_j)}$$

- $I(\cdot)$ is the brightness image ('map') of interest
- $\mathbf{s}_\ell$ is the unit direction vector of the ℓ-th source (assuming discrete source model)
- V_{ijk} is the measured correlation $(\mathbf{R}_k)_{ij}$ between antennas i and j at time t_k

Classical Fourier-based imaging

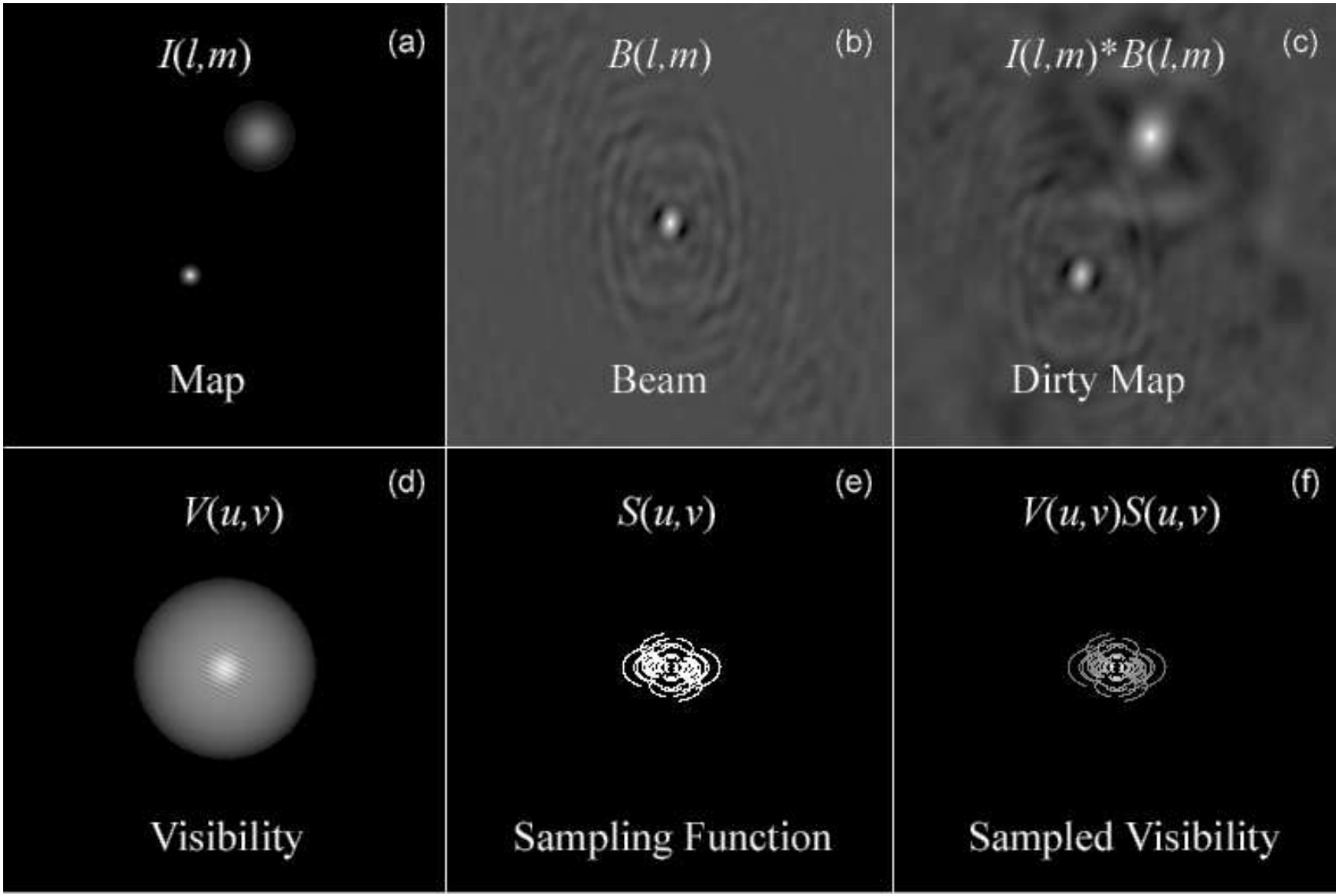
Given many samples of V_{ijk}, we can compute $I(\mathbf{s}_\ell)$ via an inverse Fourier transform:

"dirty image" : $$I_D(\mathbf{s}) := \sum_{i,j,k} V_{ijk}\, e^{j\,\mathbf{s}^T(\mathbf{r}_i - \mathbf{r}_j)} =: \sum_{\ell} I(\mathbf{s}_\ell)\, B(\mathbf{s} - \mathbf{s}_\ell) = I * B$$

"dirty beam" : $$B(\mathbf{s} - \mathbf{s}_\ell) := \sum_{i,j,k} e^{j\,(\mathbf{s}-\mathbf{s}_\ell)^T(\mathbf{r}_i - \mathbf{r}_j)}$$

Every point source excites a beam $B(\cdot)$ centered at its location $\mathbf{s}_\ell$

Imaging—the dirty image



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Imaging—adaptive beamforming techniques

Matrix model

- Recall

$$(\mathbf{R}_k)_{ij} \equiv V_{ijk} = \sum_{\ell=1}^{d} I(\mathbf{s}_\ell)\, e^{-j\,\mathbf{s}_\ell^T(\mathbf{r}_i-\mathbf{r}_j)} = \sum_{\ell=1}^{d} e^{-j\,\mathbf{s}_\ell^T\mathbf{r}_i} \cdot I(\mathbf{s}_\ell) \cdot e^{j\,\mathbf{s}_\ell^T\mathbf{r}_j}$$

- In standard array signal processing notation, we have $\mathbf{R}_k = \mathbf{A}_k\,\mathbf{B}\,\mathbf{A}_k^{\mathrm{H}}$

  where

$$\mathbf{A}_k = [\mathbf{a}_k(\mathbf{s}_1), \cdots, \mathbf{a}_k(\mathbf{s}_d)], \qquad \mathbf{B} = \begin{bmatrix} I(\mathbf{s}_1) & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & I(\mathbf{s}_d) \end{bmatrix}$$

- $\mathbf{a}_k(\mathbf{s}) = \begin{bmatrix} e^{-j\mathbf{s}^T\mathbf{r}_1(t_k)} \\ \vdots \\ e^{-j\mathbf{s}^T\mathbf{r}_p(t_k)} \end{bmatrix}$ is the array response vector.

Imaging—adaptive beamforming techniques

Classical imaging

$$I_D(\mathbf{s}) = \sum_{i,j,k} V_{ijk} e^{j\mathbf{s}^T\mathbf{r}_i} \cdot e^{-j\mathbf{s}^T\mathbf{r}_j} = \sum_{i,j,k} (\mathbf{R}_k)_{ij} (\bar{\mathbf{a}}_k(\mathbf{s}))_i (\mathbf{a}_k(\mathbf{s}))_j = \sum_k \mathbf{a}_k^{\mathrm{H}}(\mathbf{s}) \mathbf{R}_k \mathbf{a}_k(\mathbf{s})$$

Minimum Variance Distortionless Response (MVDR) imaging

- Pseudo-spectrum: $I'_D(\mathbf{s}) := \sum_k \mathbf{w}_k^{\mathrm{H}}(\mathbf{s}) \mathbf{R}_k \mathbf{w}_k(\mathbf{s})$
- MVDR criterium:

$$\hat{\mathbf{w}}_k(\mathbf{s}) = \arg\min_{\mathbf{w}_k} \mathbf{w}_k^{\mathrm{H}} \hat{\mathbf{R}}_k \mathbf{w}_k \qquad \text{such that} \qquad \mathbf{w}_k^{\mathrm{H}} \mathbf{a}_k(\mathbf{s}) = 1$$

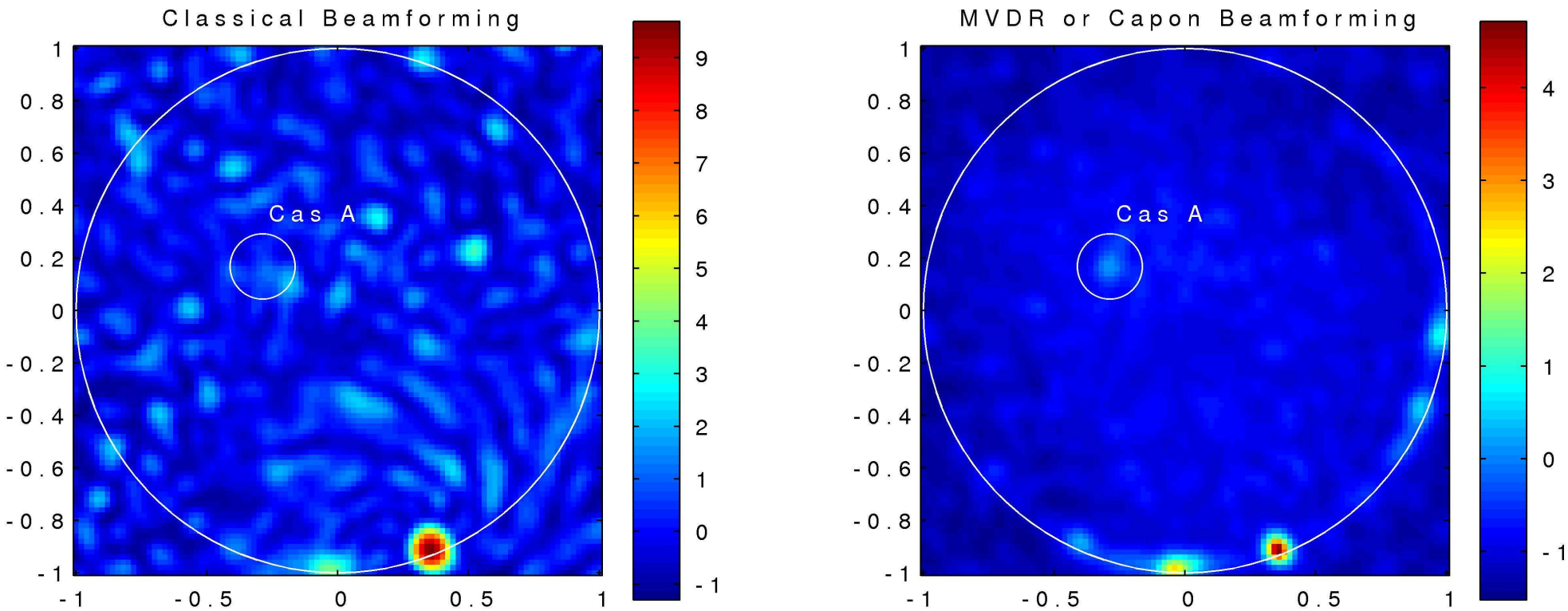
- The solution to this problem is

$$\hat{\mathbf{w}}_k = \beta_k \hat{\mathbf{R}}_k^{-1} \mathbf{a}_k(\mathbf{s}), \qquad \text{where} \qquad \beta_k = \frac{1}{\mathbf{a}_k^{\mathrm{H}}(\mathbf{s}) \hat{\mathbf{R}}_k^{-1} \mathbf{a}_k(\mathbf{s})}$$

$$\Rightarrow \qquad I'_D(\mathbf{s}) = \sum_{k=1}^{K} \frac{1}{\mathbf{a}_k^{\mathrm{H}}(\mathbf{s}) \hat{\mathbf{R}}_k^{-1} \mathbf{a}_k(\mathbf{s})}$$

Imaging—“dirty images” from the LOFAR test station

Left: **Classical beamforming;** ***right:*** **MVDR**



- Snapshot image showing Cassiopeia A along with interference at the horizon

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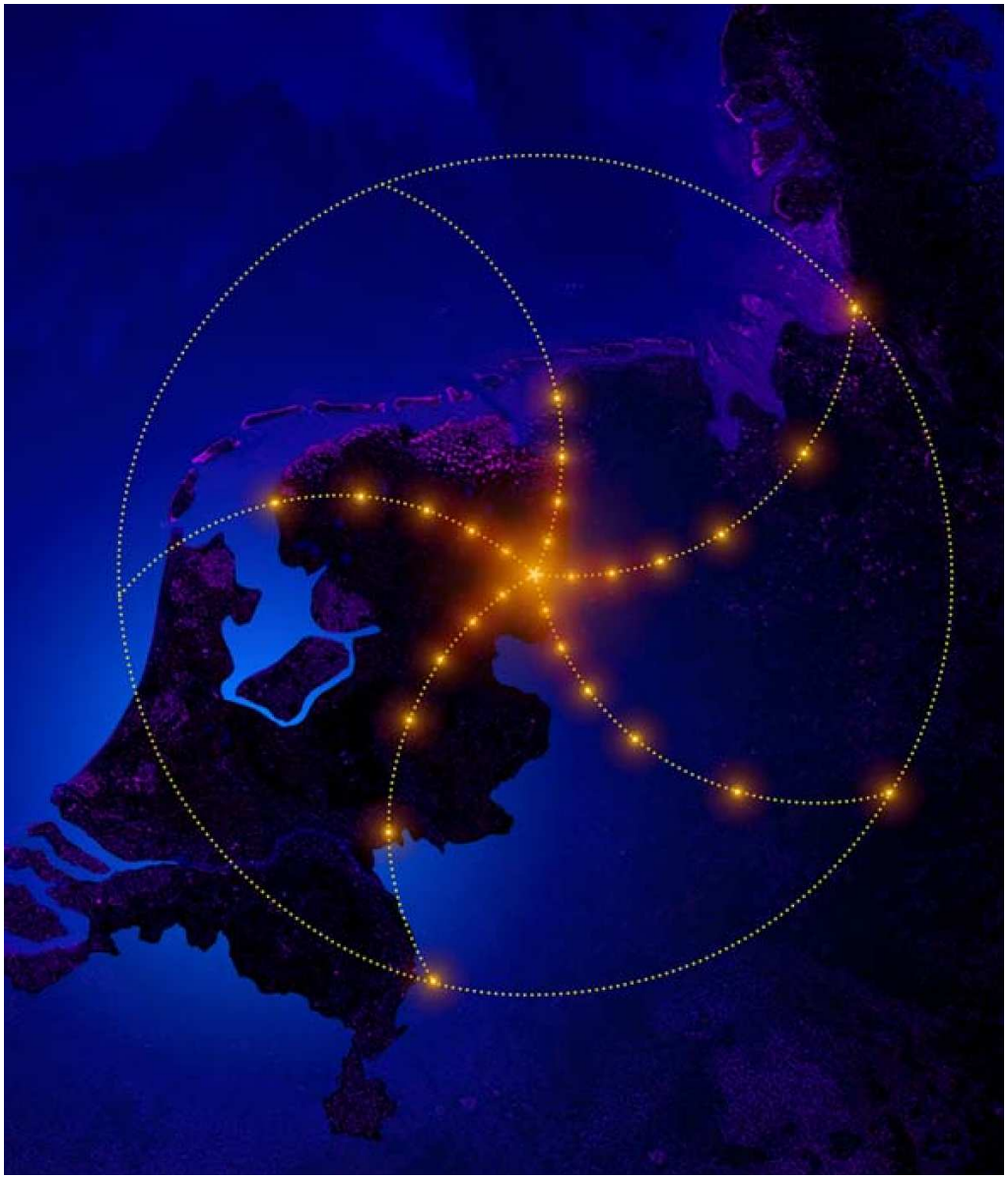
The LOFAR design

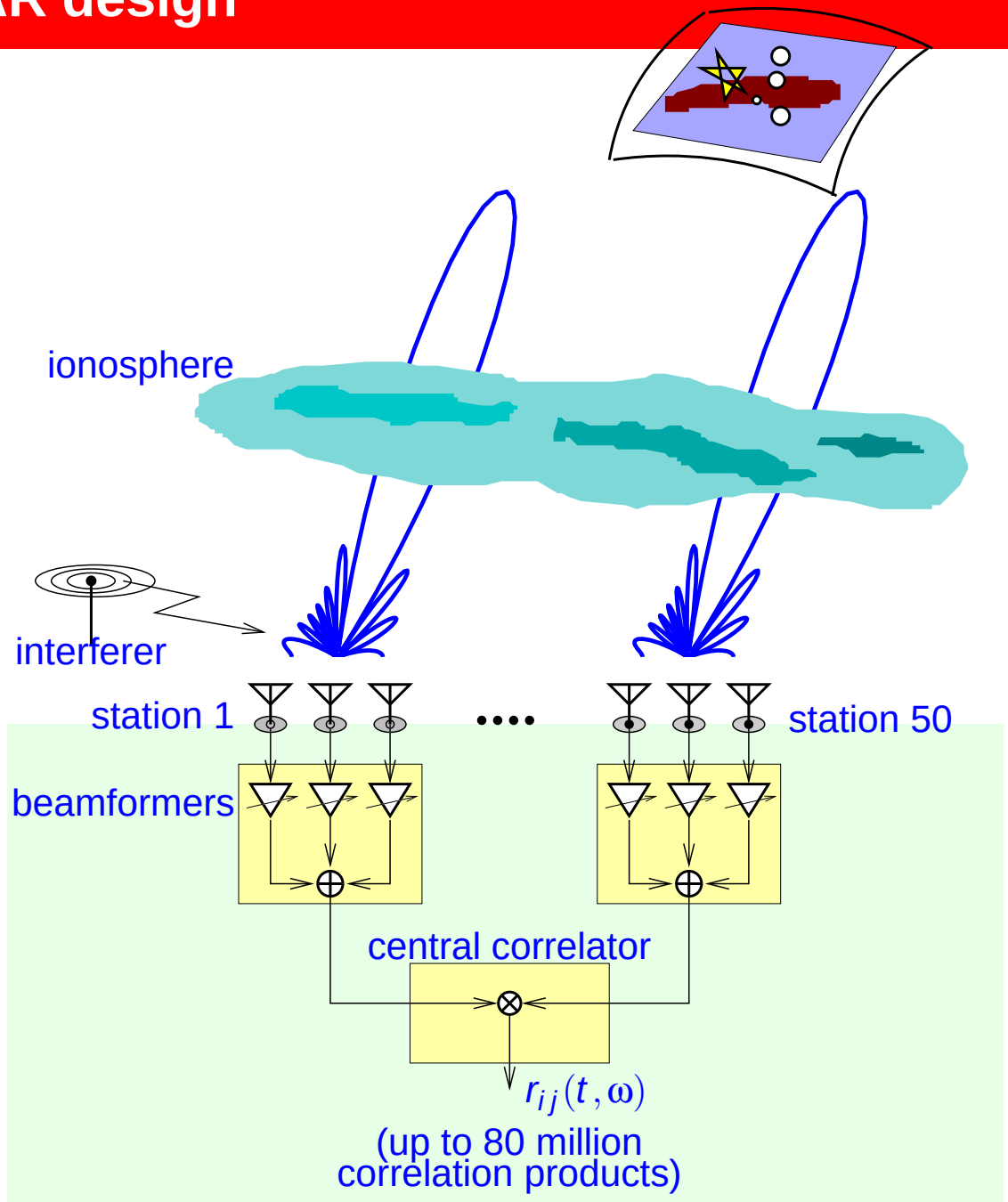
Low Frequency Array

- 77 stations (Netherlands, Germany)
- 200 elements/station
- dense core, 3000 elements
- 20–240 MHz, resolution 1 kHz
- 8 simultaneous beams, each 4 MHz
- operational in 2007 (?)



The LOFAR design

LOFAR setup



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The central processor

IBM BlueGene/L

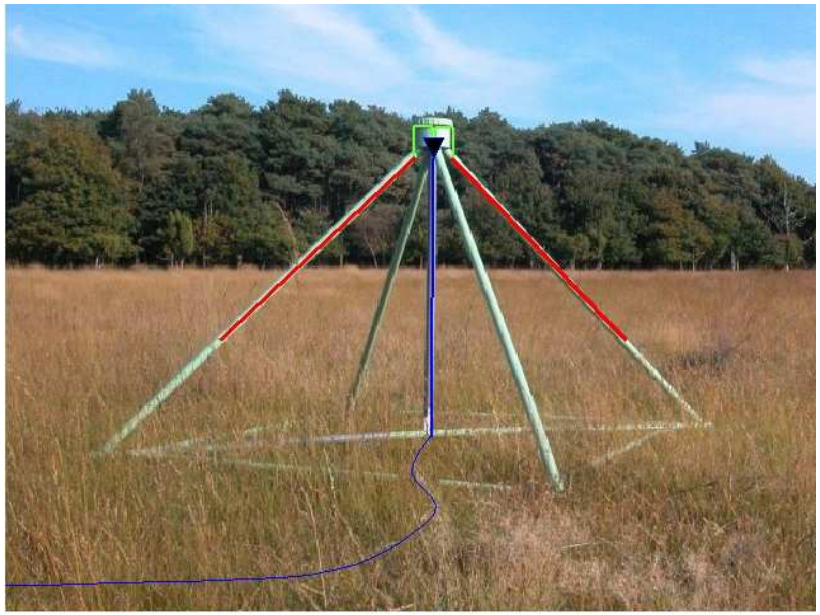
- 64k compute nodes, 180–360 TFlops

  Antennas produce 20 Tbps.

- Data is stored for 10 s, correlated, then discarded

The LOFAR design

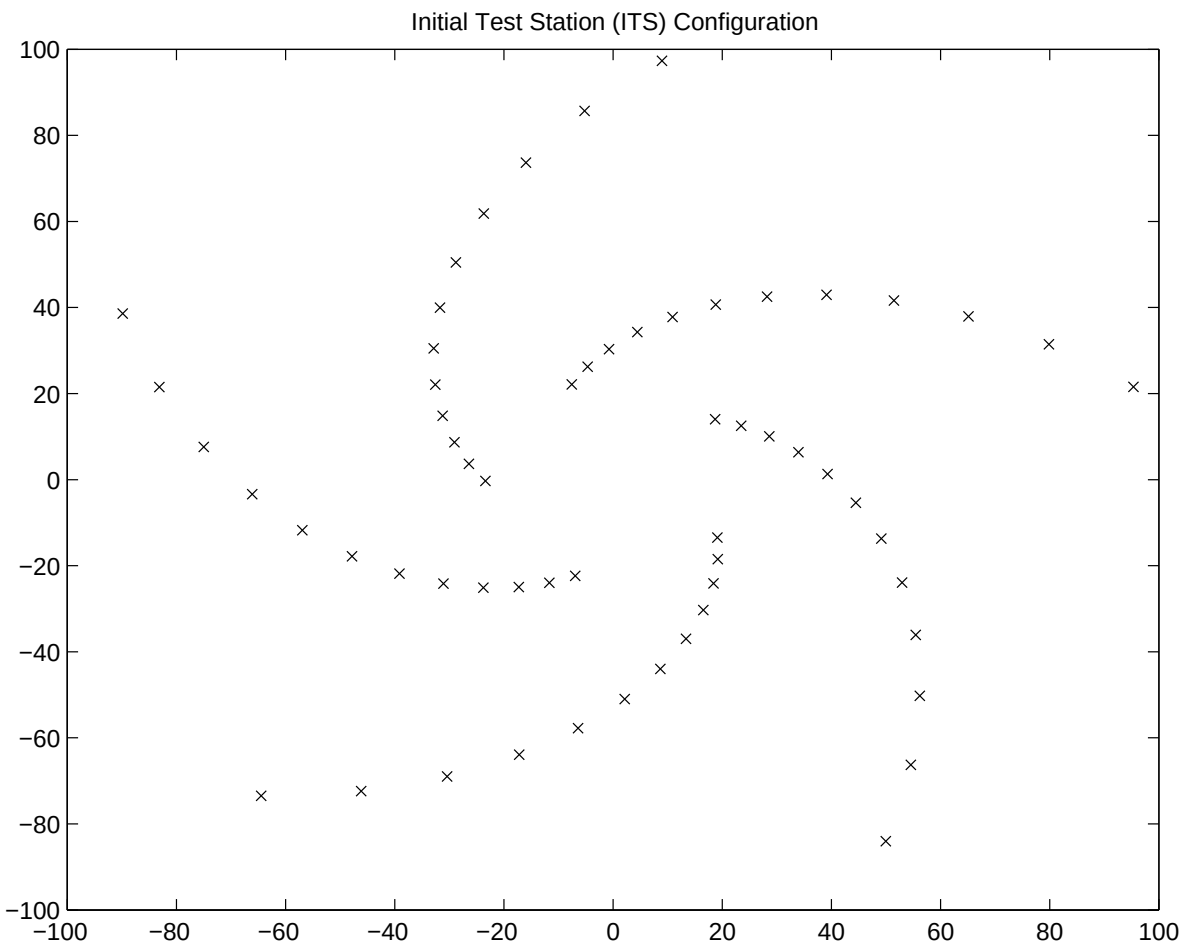
LOFAR antenna (2 dipoles)



Initial Test Station (60 elements)

LOFAR stations

Station Configuration



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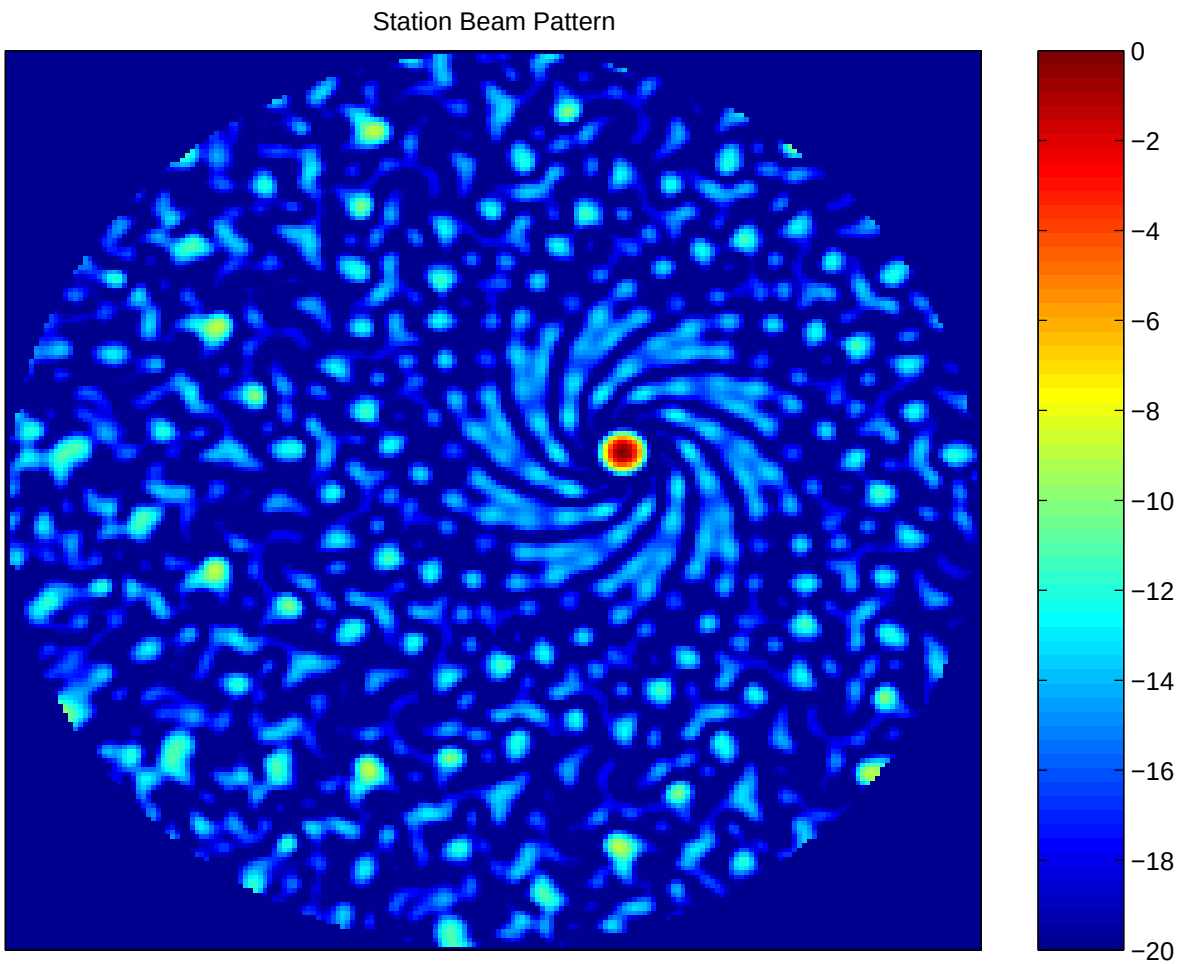
LOFAR stations

Station Beam



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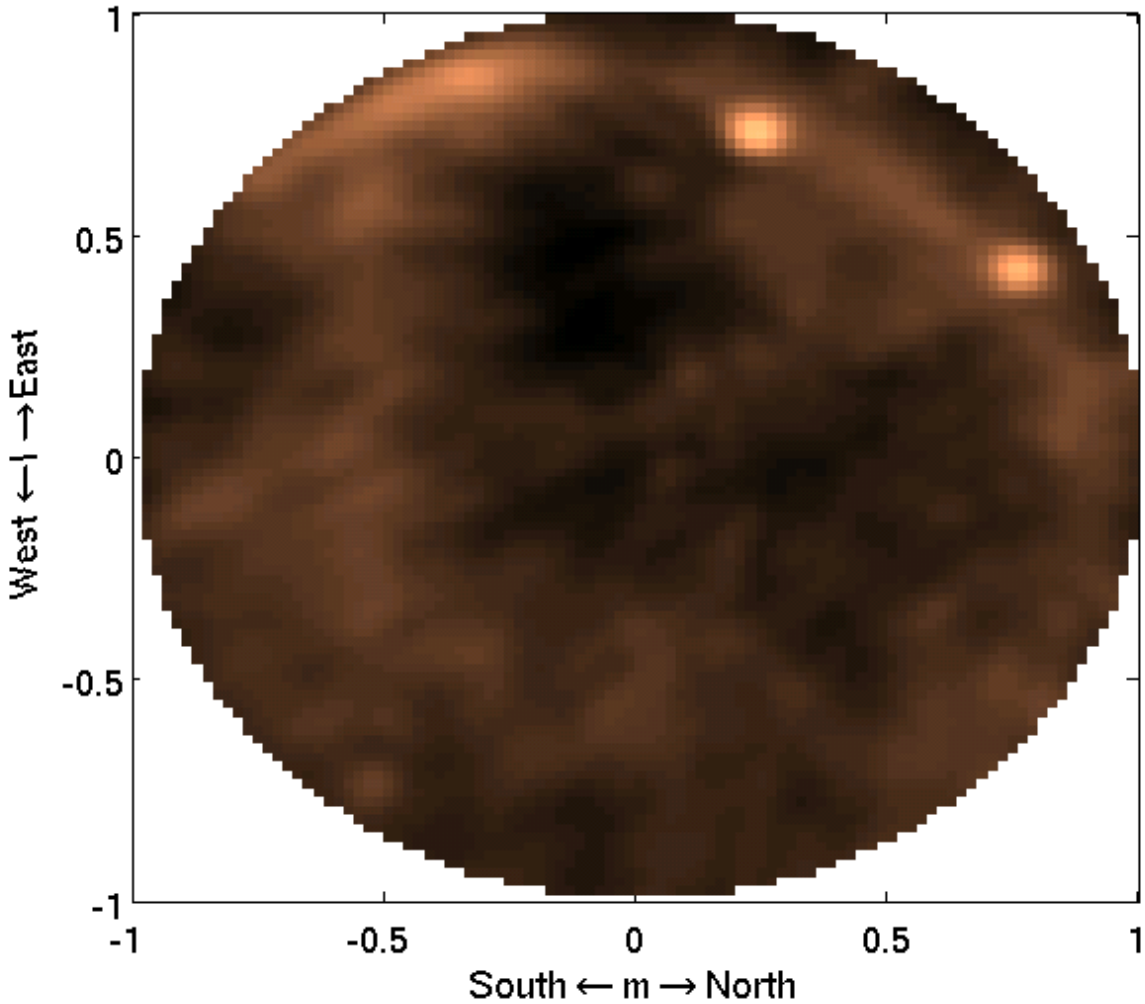
The LOFAR design

Core Station 1 (64 elements)

First LOFAR images

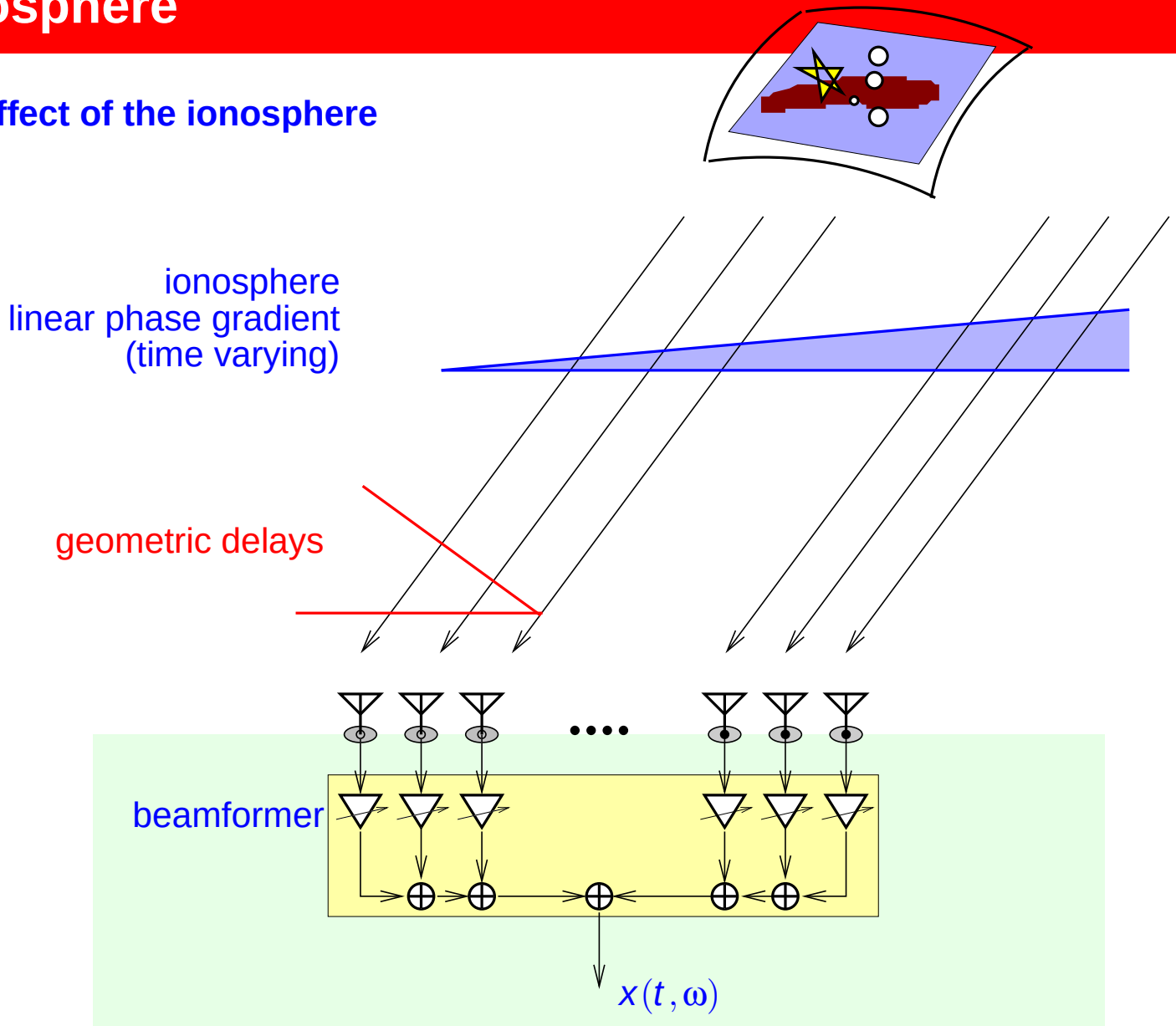


First image of ”Core Station 1” (21 Sept 2006)

48 antennas, 57 MHz, averaging 43 subbands of 156 kHz, over 512 seconds

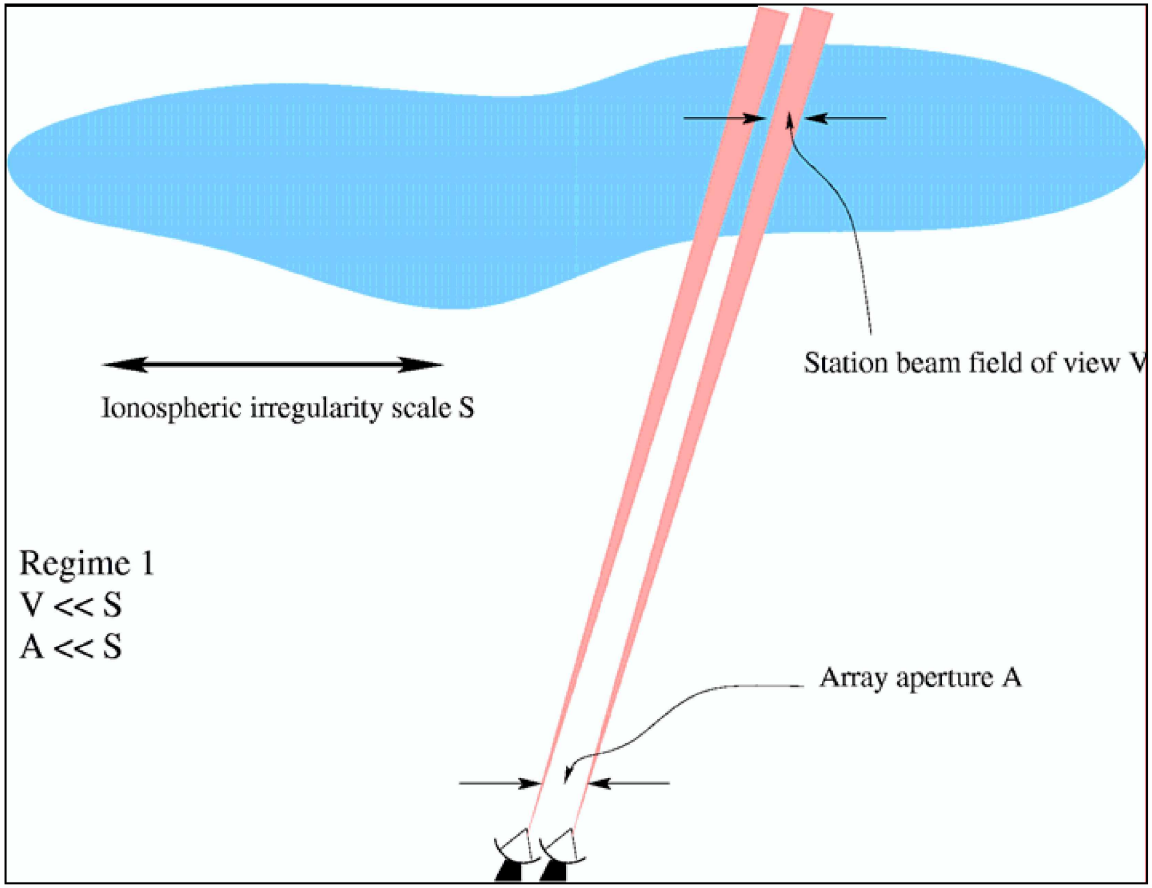
Ionosphere

The effect of the ionosphere



(movie of Virgo A observed by the VLA at 74 MHz)

Regime 1



- Existing telescopes at higher frequencies. Can assume 1 calibrator in beam
- Ionosphere phases factor out, estimate only the antenna gains and noise powers.

Calibration–regime 1

- Collect telescope outputs in vector $\mathbf{x}(t) = \begin{bmatrix} x_1(t) \\ \vdots \\ x_M(t) \end{bmatrix}$, gains in $\mathbf{g}(t) = \begin{bmatrix} g_1(t) \\ \vdots \\ g_M(t) \end{bmatrix}$,

  noise powers in $\mathbf{D} := \mathrm{E}\{\mathbf{n}(t)\mathbf{n}(t)^{\mathrm{H}}\} = \begin{bmatrix} d_1 & & \\ & \ddots & \\ & & d_M \end{bmatrix}$.

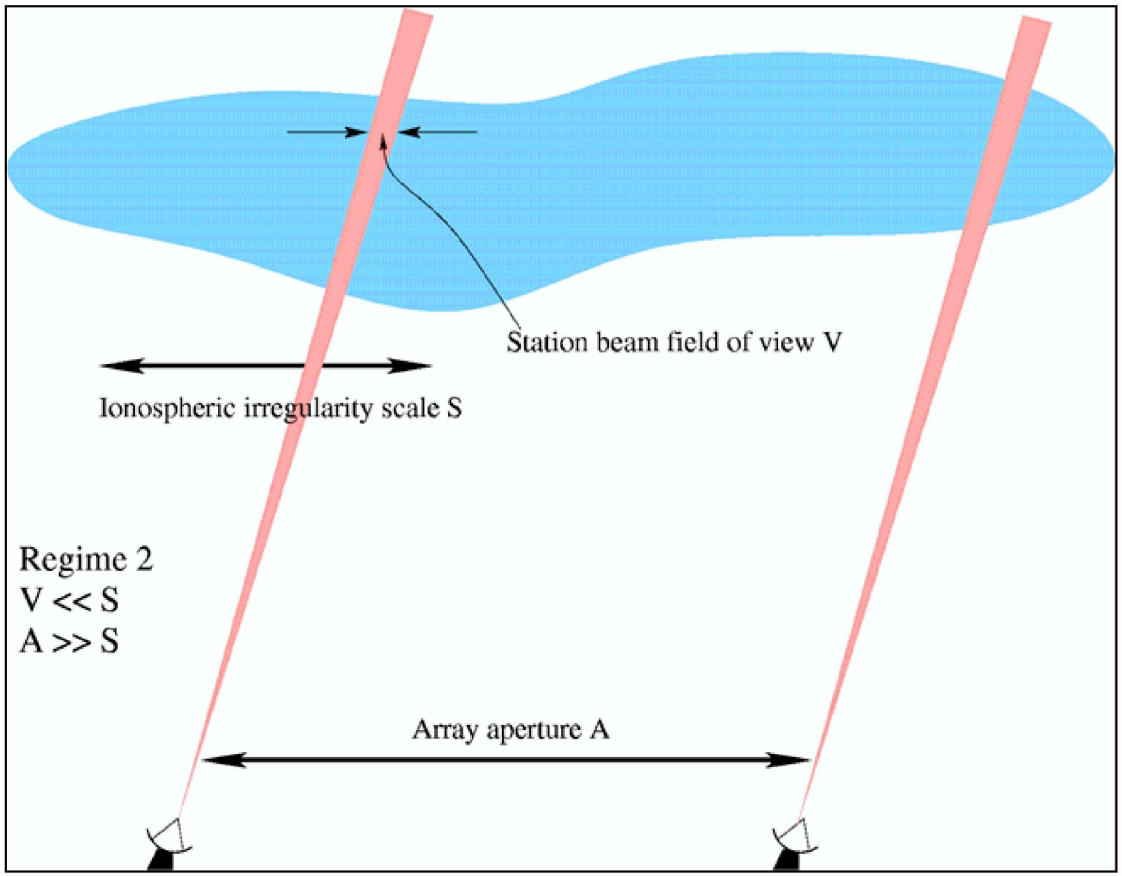
- Compute the sample covariance matrix $\hat{\mathbf{R}}(t)$, with model

$$\mathbf{R}(t) = \mathrm{E}\{\mathbf{x}(t)\mathbf{x}(t)^H\} \;=\; \mathbf{g}\mathrm{E}\{|s(t)|^2\}\mathbf{g}^{\mathrm{H}} \;+\; \mathrm{E}\{\mathbf{n}(t)\mathbf{n}(t)^{\mathrm{H}}\}$$

- Source power $\sigma_s = \mathrm{E}\{|s(t)|^2\}$ is known. Normalize to $\sigma_s = 1$.

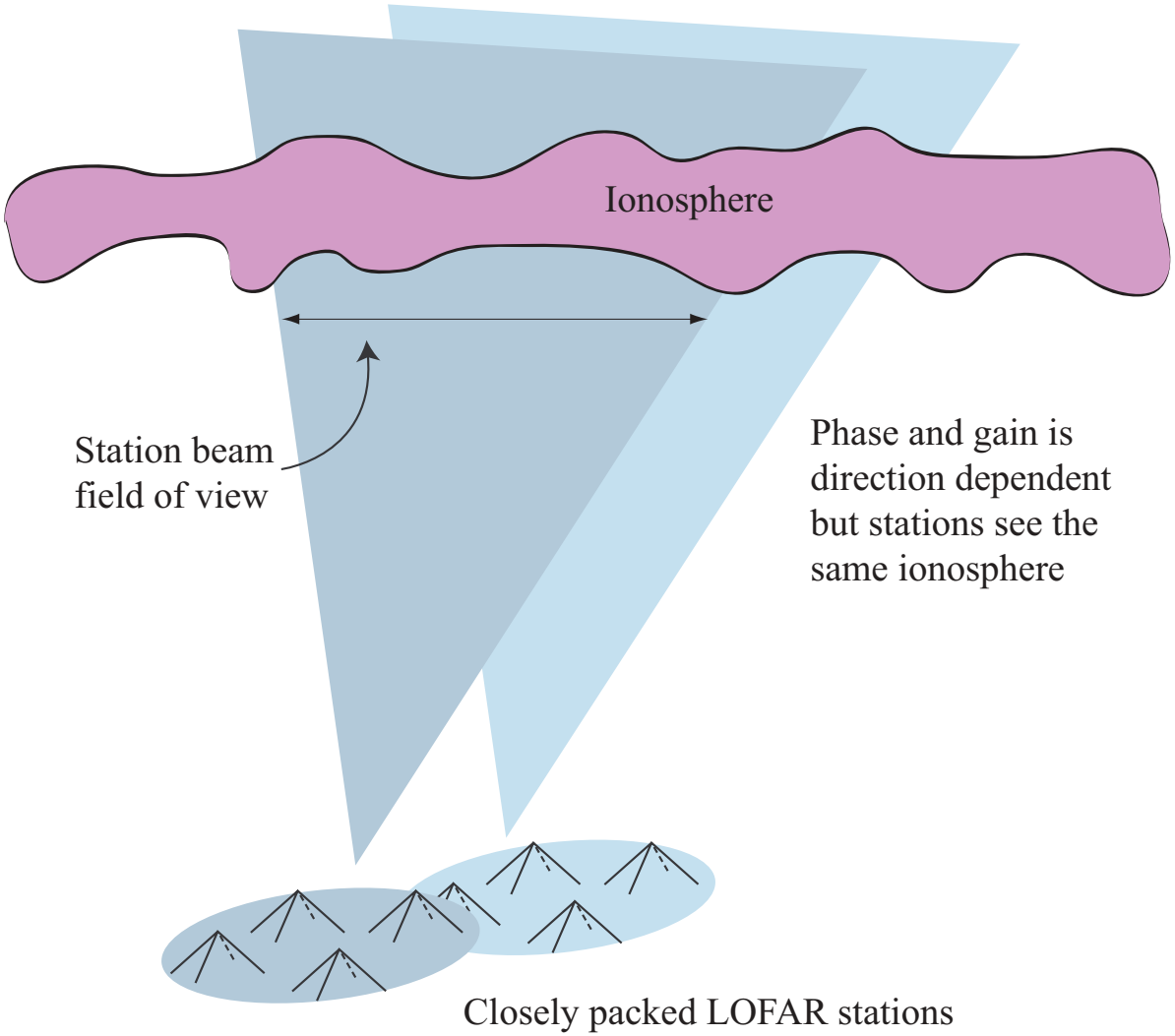
$$\mathbf{R} \;=\; \mathbf{g}\mathbf{g}^H \;+\; \mathbf{D} \qquad \text{(rank-1 factor analysis model)}$$

Regime 2



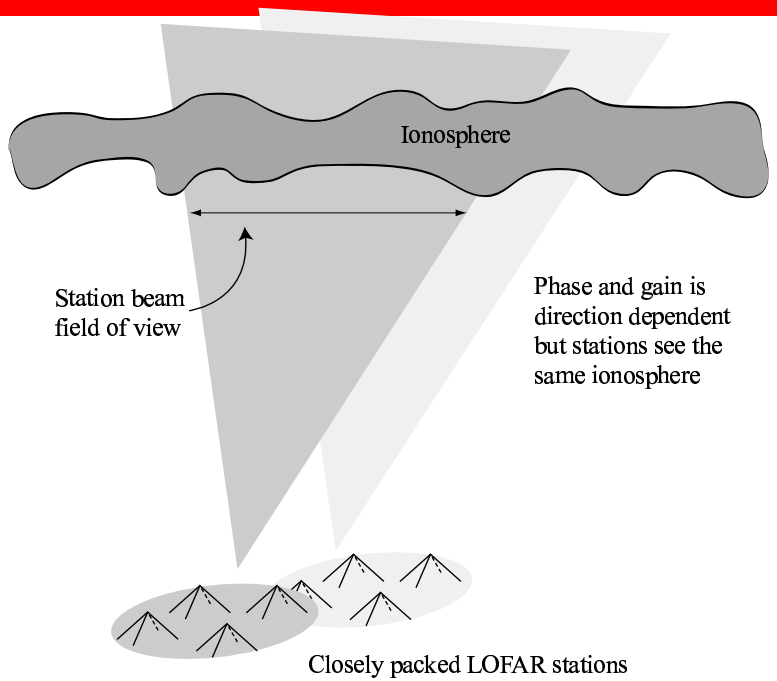
- Centimeter wave observation, VLBI. Still single source. Can give defocusing.
- Can assign ionospheric phases to antennas, and use the same algorithm as before for telescope gain estimation. Need algorithm for defocusing

Regime 3



- Existing compact low frequency arrays, individual stations, or the central core of LOFAR. A LOFAR station beam ~ 5°, sidelobe level –13 dB. Coherent image.

Calibration–regime 3



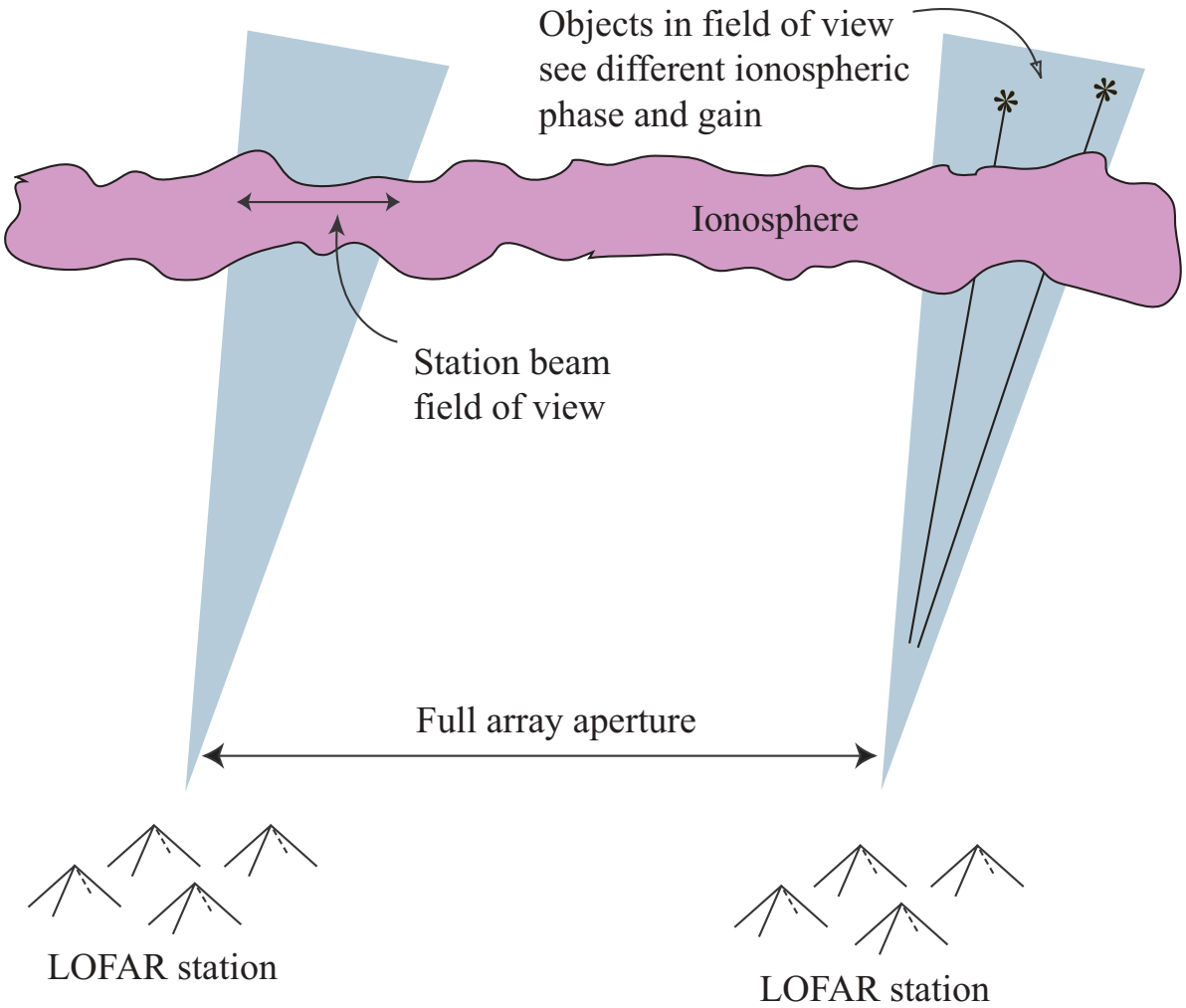
In regime 3, the unknown ionospheric phase is not observable in the correlations (can assign it to sources).

- Instead of 1 calibrator source, several will be visible (say Q):

$$\mathbf{R} = \mathbf{G}\mathbf{R}_0\mathbf{G}^H + \mathbf{D}$$

with $\mathbf{R}_0 = \mathbf{K}\Sigma_s\mathbf{K}^{\mathrm{H}}$ ($M \times M$ rank Q) known, $\mathbf{K}$ is array response due to geometry (Fourier vectors), and $\mathbf{G} = \mathrm{diag}(\mathbf{g})$: direction-indept gain. Solvable if $Q < M$.

Regime 4



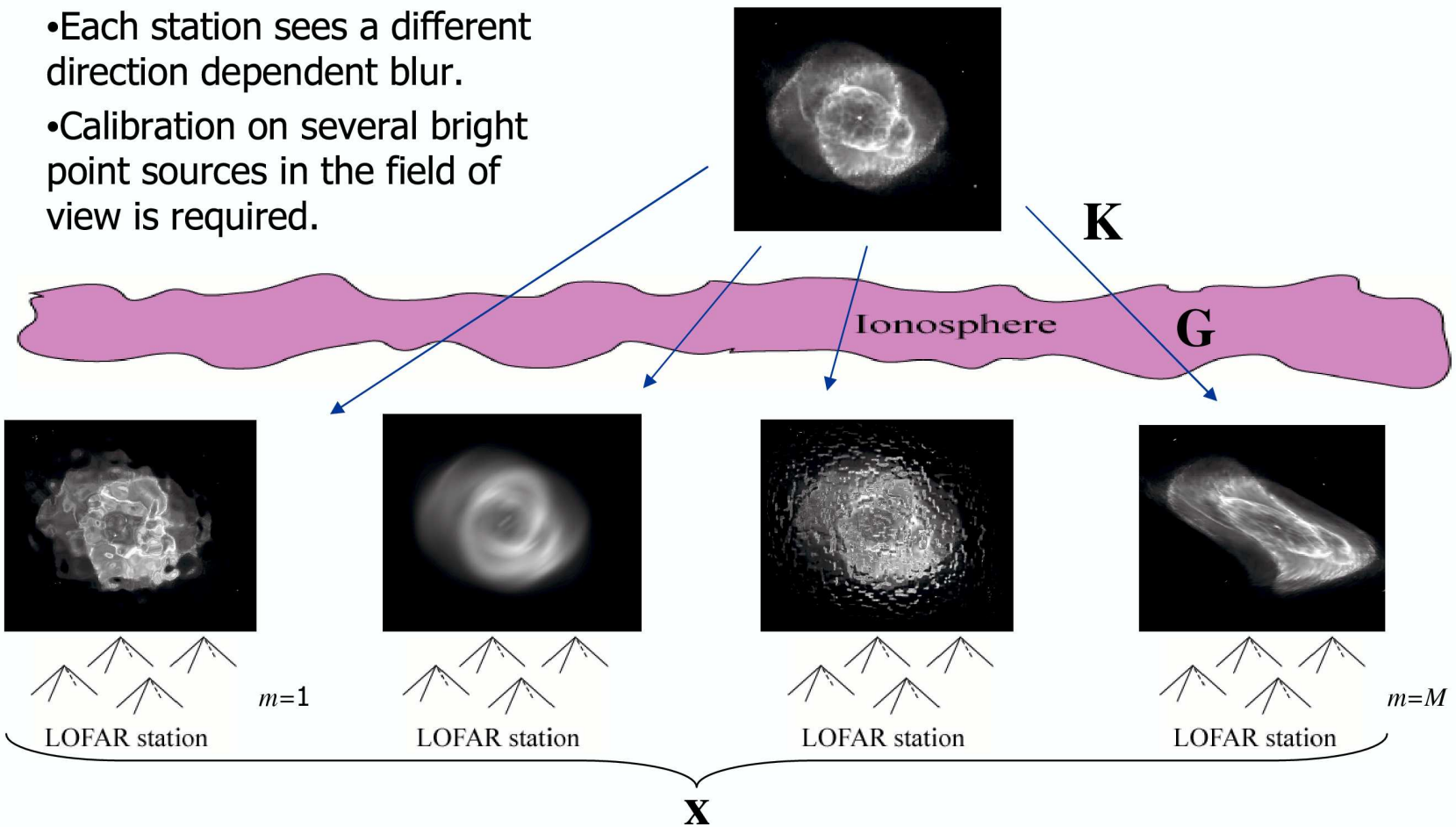
- The complete LOFAR system: requires direction dependent calibration

Is this identifiable?

Calibration–regime 4

Direction-dependent calibration

- Each station sees a different direction dependent blur.
- Calibration on several bright point sources in the field of view is required.

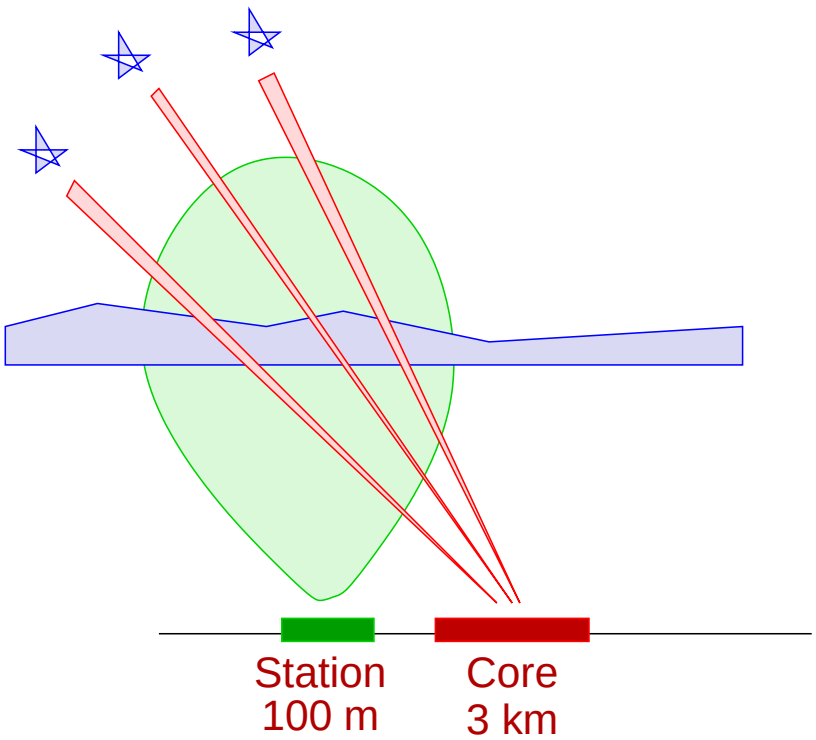


Calibration–regime 4

Calibrated subarray (central core)

The LOFAR design contains a central core of 32 stations. A possible algorithm is

- Calibrate the subarray separately (regime 3).
- Use it to sample the other station beams (correlate core with stations). This suppresses most of the other sources ⇒ single source calibration

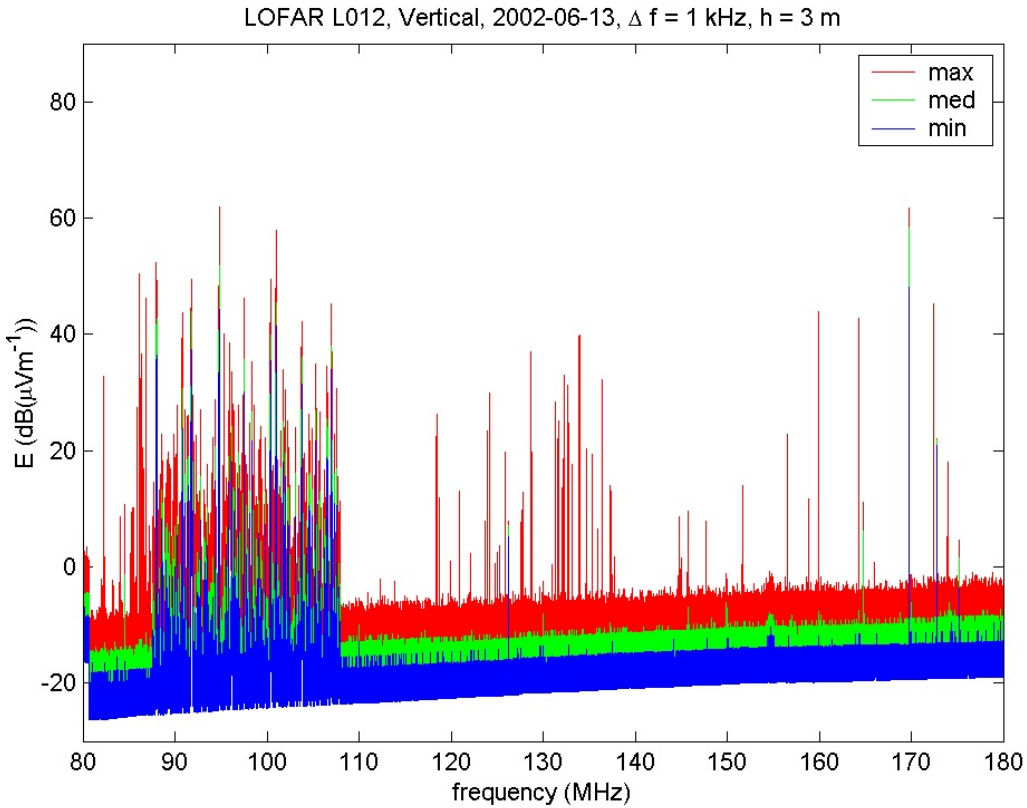


Ongoing research

- Statistical model of the ionosphere (model error, optimal basis, tracking)
- Can we reach the required dynamic range (70 dB)?
- How do errors show up in the final image?
- RFI mitigation

Interference

LOFAR RFI situation



- Adaptive nulling (station/central), frequency notches, subtraction
- Station processing should maintain smooth behavior of beamshapes

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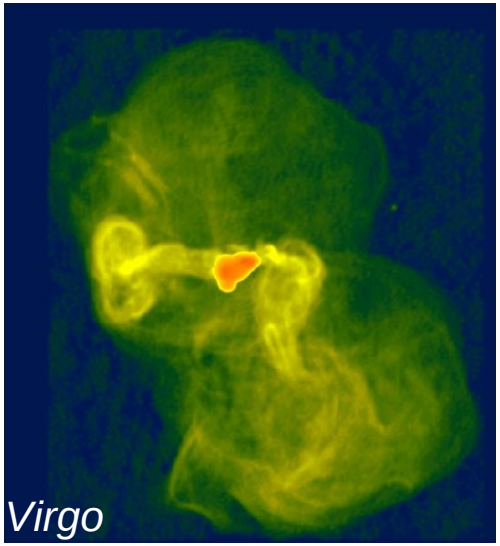
Conclusions

Radio astronomy is an interesting application area for Array Signal Processing:

- Imaging and deconvolution
- Interference reduction using subspace techniques
- Calibration



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