Deterministic Blind Modulation-Induced Source Separation for Digital Wireless Communications

Geert Leus, Piet Vandaele, and Marc Moonen

***Abstract*—In this paper, we present a new simple deterministic blind source separation algorithm, which is based on modulating the same data symbol sequence with different code sequences and transmitting the resulting modulated data symbol sequences through different antennas. The algorithm does not exploit the finite alphabet property of the data symbols. As a result, no iterations are required, and convergence is not an issue. Instantaneous mixtures (frequency-flat fading), as well as convolutive mixtures (frequency-selective fading), can be handled. In the case of a convolutive mixture, the difficulties that occur when the users have unequal channel orders are avoided. Moreover, the proposed algorithm is robust against channel order underestimation.**

***Index Terms*—Blind source separation, communications, convolutive mixtures, instantaneous mixtures, transmit diversity.**

I. Introduction

THE BLIND separation of different digital signals, of which only an instantaneous (frequency-flat fading) or convolutive (frequency-selective fading) mixture is observed, is considered here. Compared with *stochastic* blind algorithms, *deterministic* blind algorithms can be applied on much smaller blocks of received samples. Therefore, we will focus on *deterministic* blind source separation in this work.

For an *instantaneous mixture*, several deterministic blind source separation algorithms have already been presented. A well-known iterative algorithm that exploits the finite alphabet property of the digital signals is the iterative least squares algorithm with projection (ILSP) [1]. However, this algorithm does not necessarily converge to the global minimum. Hence, to find the actual global minimum, the ILSP algorithm requires several random initializations or an initialization based on a noniterative algorithm (see below). Another iterative algorithm that exploits the finite alphabet property of the digital signals is the hypercube algorithm [2]. This algorithm, which sequentially estimates each signal, is less complex than the ILSP algorithm. However, like the ILSP algorithm, it does not necessarily converge to the global minimum. Interesting noniterative algorithms are the analytical constant modulus algorithm (ACMA) [3] for constant modulus constellations and the real analytical constant modulus algorithm (RACMA) [4] and the algorithm presented in [5] for a BPSK constellation. Although near-optimum, these approaches are computationally expensive. Finally, a simple recursive noniterative algorithm for a BPSK constellation can be found in [6].

In addition, for a *convolutive mixture*, some deterministic blind source separation algorithms have already been presented. Extensions of the ILSP algorithm [1] and the hypercube algorithm [2] to convolutive mixtures can be found in [7] and [8], respectively. In addition, the subspace intersection (SSI) algorithms presented in [9] and [10] are very popular. When the users have equal channel orders, these algorithms consist of two steps. First, the convolutive mixture is transformed into an instantaneous mixture using a direct blind symbol estimation approach (only an instantaneous mixture of the digital signals is identified). Note, however, that this can also be done by using a blind channel estimation approach (only an instantaneous mixture of the channels is identified) followed by a channel inversion, as mentioned in [9] (see [11] and [12] for an extensive treatment of deterministic blind channel estimation in a multiuser system). Next, one of the above algorithms for instantaneous mixtures is used. When the users have unequal channel orders, difficulties occur, and a cumbersome iterative procedure is required. The major drawback of the SSI algorithms presented in [9] and [10] is that they are rather sensitive to channel order mismatch.

In this paper, we show that by modulating the same data symbol sequence with different code sequences and transmitting the resulting modulated data symbol sequences through different antennas, we can develop a new simple deterministic blind source separation algorithm. This algorithm does not exploit the finite alphabet property of the data symbols. As a result, no iterations are required, and convergence is not an issue. Instantaneous mixtures (frequency-flat fading), as well as convolutive mixtures (frequency-selective fading), can be handled. In the case of a convolutive mixture, the difficulties that occur when the users have unequal channel orders are avoided. Moreover, the proposed algorithm is robust against channel order underestimation.

The idea of modulating a data symbol sequence with a code sequence is not new. In [13] and [14], it is used to get rid of the identifiability conditions for second-order blind channel estimation in a single-user system. We use it, on the other hand, to solve the source separation problem. Moreover, the algorithms presented in [13] and [14] are stochastic, whereas the algorithm we

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G. Leus and M. Moonen are with the Department of Electrical Engineering, Katholieke Universiteit Leuven, Heverlee, Belgium (e-mail: geert.leus@esat.kuleuven.ac.be; marc.moonen@esat.kuleuven.ac.be).

P. Vandaele is with the Corporate Research Center, Alcatel Telecom, Antwerpen, Belgium (e-mail: piet.vandaele@alcatel.be).

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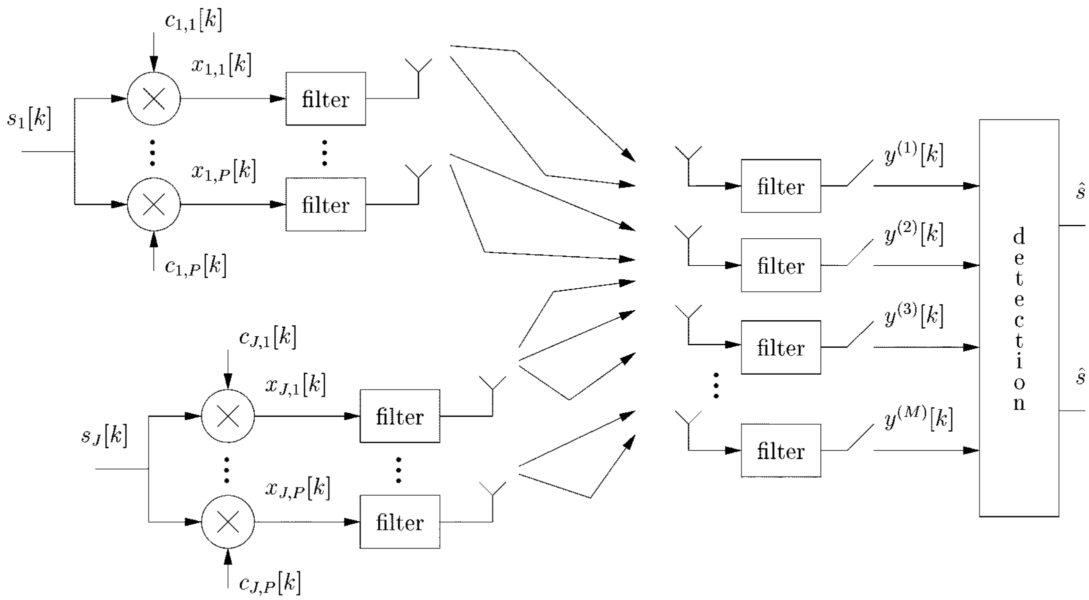

Fig. 1. Multiuser system based on code modulation.

develop is deterministic. Of course, there also exist other types of coding that do not decrease the information rate. In [15], for example, correlative coding is used to solve the source separation problem. However, this algorithm is rather complex and, like the algorithms presented in [13] and [14], it is stochastic.

In Section II, we introduce the data model. In Section III, we then state the source separation problem under consideration. The proposed deterministic blind source separation algorithm is presented in Section IV. Simulation results are given in Section V. We end with some conclusions in Section VI.

II. Data Model

We first introduce some basic notation. We use lower-case boldface letters to denote vectors and upper-case boldface letters to denote matrices. In addition

- $(\cdot)^T$ transpose;
- $(\cdot)^H$ Hermitian transpose;
- $|\cdot|$ absolute value;
- $||\cdot||$ Frobenius norm.

Let us then consider a system of J users and M (base station) receive antennas, where each user is transmitting through P transmit antennas (see Fig. 1). At the pth transmit antenna ($p = 1, 2, \cdots, P$), the jth user ($j = 1, 2, \cdots, J$) modulates his data symbol sequence $s_j[k]$ (with data symbols in some finite alphabet Ω) with the code sequence $c_{j,p}[k]$, leading to the following modulated data symbol sequence:[1]

$$x_{j,p}[k] = s_j[k]c_{j,p}[k]. \tag{1}$$

To avoid introducing (additional) modulus variations, we assume that the code sequences $\{\{c_{j,p}[k]\}_{j=1}^J\}_{p=1}^P$ are constant modulus with modulus 1:

$$|c_{j,p}[k]| = 1, \quad \text{for } j = 1, 2, \cdots, J \text{ and } p = 1, 2, \cdots, P. \tag{2}$$

[1]In the DS-CDMA jargon, this means that we use a spreading factor of 1.

The modulated data symbol sequence $x_{j,p}[k]$ is then transmitted through the pth transmit antenna at the data symbol rate $1/T$, where T is the data symbol period. Next, if we sample the M receive antennas at the data symbol rate $1/T$, the received sequence at the mth receive antenna ($m = 1, 2, \cdots, M$) is given by

$$y^{(m)}[k] = \sum_{j=1}^{J}\sum_{p=1}^{P}\sum_{k'=-\infty}^{+\infty} g_{j,p}^{(m)}[k-k']x_{j,p}[k'] + e^{(m)}[k]$$

where $e^{(m)}[k]$ is the discrete-time additive noise at the mth receive antenna, and $g_{j,p}^{(m)}[k]$ is the discrete-time channel from the pth transmit antenna of the jth user to the mth receive antenna, including the transmit and receive filters. Stacking the received samples from the M receive antennas

$$\mathbf{y}[k] = [\, y^{(1)}[k] \quad y^{(2)}[k] \quad \cdots \quad y^{(M)}[k]\,]^T$$

we obtain

$$\mathbf{y}[k] = \sum_{j=1}^{J}\sum_{p=1}^{P}\sum_{k'=-\infty}^{+\infty} \mathbf{g}_{j,p}[k-k']x_{j,p}[k'] + \mathbf{e}[k]$$

where $\mathbf{e}[k]$ is similarly defined as $\mathbf{y}[k]$, and $\mathbf{g}_{j,p}[k]$ is the discrete-time $M \times 1$ vector channel for the pth transmit antenna of the jth user, which is given by

$$\mathbf{g}_{j,p}[k] = \left[\, g_{j,p}^{(1)}[k] \quad g_{j,p}^{(2)}[k] \quad \cdots \quad g_{j,p}^{(M)}[k]\,\right]^T.$$

Remark 1: Note that a similar data model is obtained if the spatial oversampling under consideration is replaced by or combined with temporal oversampling, i.e., sampling at a multiple of the data symbol rate. Hence, the results presented in this paper can easily be generalized for such a scenario.

We make the assumption that every vector channel from the set $\{\mathbf{g}_{j,p}[k]\}_{p=1}^P$ is an FIR vector filter of the same order L_j with the same delay index δ_j ($\mathbf{g}_{j,p}[k] \neq \mathbf{0}$ for $k = \delta_j$ and $k = \delta_j + L_j$, and $\mathbf{g}_{j,n}[k] = \mathbf{0}$ for $k < \delta_j$ and $k > \delta_j + L_j$). Although this is not strictly necessary, it simplifies the description of the proposed algorithm. We further assume w.l.o.g. that $\delta_j = 0$ for $j = 1, 2, \cdots, J$.

For a burst length of K ($s_j[0], s_j[1], \cdots, s_j[K-2]$ and $s_j[K-1]$ are the data symbols of interest for the jth user), the matrix that plays a central role in the next sections is the following $(Q+1)M \times K$ output matrix:

$$\mathbf{Y}_k = \begin{bmatrix} \mathbf{y}[k] & \mathbf{y}[k+1] & \cdots & \mathbf{y}[k+K-1] \\ \vdots & \vdots & & \vdots \\ \mathbf{y}[k+Q] & \mathbf{y}[k+Q+1] & \cdots & \mathbf{y}[k+Q+K-1] \end{bmatrix}$$

where Q determines the amount of temporal smoothing. This output matrix $\mathbf{Y}_k$ can be written as

$$\mathbf{Y}_k = \sum_{j=1}^{J} \sum_{p=1}^{P} \mathcal{G}_{j,p} \mathbf{X}_{j,p,k} + \mathbf{E}_k \tag{3}$$

where $\mathbf{E}_k$ is similarly defined as $\mathbf{Y}_k$, $\mathcal{G}_{j,p}$ is the $(Q+1)M \times r_j$ ($r_j = Q+1+L_j$) channel matrix for the pth transmit antenna of the jth user, which is given by

$$\mathcal{G}_{j,p} = \begin{bmatrix} \mathbf{g}_{j,p}[L_j] & \cdots & \mathbf{g}_{j,p}[0] & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{g}_{j,p}[L_j] & \cdots & \mathbf{g}_{j,p}[0] & \cdots & \mathbf{0} \\ & & & \ddots & & \\ \mathbf{0} & \cdots & \mathbf{0} & \mathbf{g}_{j,p}[L_j] & \cdots & \mathbf{g}_{j,p}[0] \end{bmatrix}$$

and $\mathbf{X}_{j,p,k}$ is the $r_j \times K$ input matrix for the pth transmit antenna of the jth user, which is shown in (4) at the bottom of the page. Note that (3) can also be written as

$$\mathbf{Y}_k = \mathcal{G}\mathbf{X}_k + \mathbf{E}_k \tag{5}$$

where $\mathcal{G}$ is the $(Q+1)M \times r$ ($r = P\sum_{j=1}^{J} r_j$) channel matrix, which is given by

$$\mathcal{G} = [\, \mathcal{G}_{1,1} \quad \cdots \quad \mathcal{G}_{1,P} \,|\cdots|\, \mathcal{G}_{J,1} \quad \cdots \quad \mathcal{G}_{J,P} \,]$$

and $\mathbf{X}_k$ is the $r \times K$ input matrix, which is given by

$$\mathbf{X}_k = \left[\mathbf{X}_{1,1,k}^T \ \cdots \ \mathbf{X}_{1,P,k}^T \,\middle|\cdots\middle|\, \mathbf{X}_{J,1,k}^T \ \cdots \ \mathbf{X}_{J,P,k}^T \right]^T. \tag{6}$$

III. Problem Statement

For a burst length of K ($s_j[0], s_j[1], \cdots, s_j[K-2]$ and $s_j[K-1]$ are the data symbols of interest for the jth user), let us define

$$\mathbf{s}_j = [\, s_j[0] \quad s_j[1] \quad \cdots \quad s_j[K-1] \,]. \tag{7}$$

Using (1), we then know that $\mathbf{x}_{j,p}$ given by

$$\mathbf{x}_{j,p} = [\, x_{j,p}[0] \quad x_{j,p}[1] \quad \cdots \quad x_{j,p}[K-1] \,]$$

can be written as a function of $\mathbf{s}_j$, shown in (8) at the bottom of the page, where $\mathbf{C}_{j,p}$ is the $K \times K$ code matrix for the pth transmit antenna of the jth user, which is given by

$$\mathbf{C}_{j,p} = \begin{bmatrix} c_{j,p}[0] & c_{j,p}[1] & \\ & \ddots & \\ & & c_{j,p}[K-1] \end{bmatrix}. \tag{9}$$

From (4) and (6), it is then clear that every vector from the set $\{\mathbf{x}_{j,p} = \mathbf{s}_j \mathbf{C}_{j,p}\}_{p=1}^{P}$ is a row of every input matrix from the set $\{\mathbf{X}_k\}_{k=-Q}^{L_j}$ and is therefore "contained" in every output matrix from the set $\{\mathbf{Y}_k\}_{k=-Q}^{L_j}$ [see (5)]. The problem addressed here is to compute the vector $\mathbf{s}_j$ from the set $\{\mathbf{Y}_k\}_{k=A_1}^{A_2}$ with

$$-Q \le A_1 \le A_2 \le L_j \tag{10}$$

based only on the knowledge of the set of code sequences $\{c_{j,p}[k]\}_{p=1}^{P}$. Note that we define A as the number of output matrices taken into account ($A = A_2 - A_1 + 1$), which means that

$$1 \le A \le Q + L_j + 1.$$

To solve this problem, we make the following rather standard assumptions.

Assumption 1: The channel matrix $\mathcal{G}$ has full column rank r (r is then called the system order).

Assumption 2: Every input matrix from the set $\{\mathbf{X}_k\}_{k=A_1}^{A_2}$ has full row rank r.

Note that Assumption 1 is equivalent with the assumption that the FIR matrix filter

$$[\, \mathbf{g}_{1,1}[k] \quad \cdots \quad \mathbf{g}_{1,P}[k] \,|\cdots|\, \mathbf{g}_{J,1}[k] \quad \cdots \quad \mathbf{g}_{J,P}[k] \,]$$

is *irreducible* and *column reduced* (see [11]) and that

$$(Q+1)M \ge r = (Q+1)JP + P\sum_{j=1}^{J} L_j.$$

$$\mathbf{X}_{j,p,k} = \begin{bmatrix} x_{j,p}[k-L_j] & x_{j,p}[k-L_j+1] & \cdots & x_{j,p}[k-L_j+K-1] \\ \vdots & \vdots & & \vdots \\ x_{j,p}[k+Q] & x_{j,p}[k+Q+1] & \cdots & x_{j,p}[k+Q+K-1] \end{bmatrix} \tag{4}$$

$$\mathbf{x}_{j,p} = [\, s_j[0]c_{j,p}[0] \quad s_j[1]c_{j,p}[1] \quad \cdots \quad s_j[K-1]c_{j,p}[K-1] \,] = \mathbf{s}_j \mathbf{C}_{j,p} \tag{8}$$

The latter indicates that we should use $M > JP$. Assumption 2 requires that

$$K \geq r = (Q+1)JP + P\sum_{j=1}^{J} L_j.$$

IV. Deterministic Blind Source Separation Algorithm

Before discussing the proposed deterministic blind source separation algorithm in detail, we explain the main idea by means of a simple example.

Example 1: We consider an instantaneous mixture ($J = 2$ and $L_1 = L_2 = 0$), $M = 6$ receive antennas and $P = 2$ transmit antennas per user and take $Q = 0$ (temporal smoothing has no use for an instantaneous mixture). Hence, we can only examine $A_1 = A_2 = 0$ for every user. Focusing on the first user, the problem under consideration then is to compute the vector

$$\mathbf{s}_1 = [\, s_1[0] \quad s_1[1] \quad \cdots \quad s_1[K-1]]$$

from $\mathbf{Y}_0$ based only on the knowledge of the set of code sequences $\{c_{1,1}[k], c_{1,2}[k]\}$. If we assume no additive noise is present, $\mathbf{Y}_0$ can be written as

$$\mathbf{Y}_0 = \begin{bmatrix} y^{(1)}[0] & y^{(1)}[1] & \cdots & y^{(1)}[K-1] \\ y^{(2)}[0] & y^{(2)}[1] & \cdots & y^{(2)}[K-1] \\ \vdots & \vdots & & \vdots \\ y^{(6)}[0] & y^{(6)}[1] & \cdots & y^{(6)}[K-1] \end{bmatrix}$$

$$= [\mathcal{G}_{1,1} \quad \mathcal{G}_{1,2} | \mathcal{G}_{2,1} \quad \mathcal{G}_{2,2}]$$

$$\cdot \left[\begin{array}{cccc} s_1[0]c_{1,1}[0] & s_1[1]c_{1,1}[1] & \cdots & s_1[K-1]c_{1,1}[K-1] \\ s_1[0]c_{1,2}[0] & s_1[1]c_{1,2}[1] & \cdots & s_1[K-1]c_{1,2}[K-1] \\ \hline s_2[0]c_{2,1}[0] & s_2[1]c_{2,1}[1] & \cdots & s_2[K-1]c_{2,1}[K-1] \\ s_2[0]c_{2,2}[0] & s_2[1]c_{2,2}[1] & \cdots & s_2[K-1]c_{2,2}[K-1] \end{array}\right].$$

The key observation then is that if we multiply $\mathbf{Y}_0$ to the right with, respectively, $\mathbf{C}_{1,1}^H$ and $\mathbf{C}_{1,2}^H$ [see (9)], the intersection of the row spaces of the obtained matrices contains the vector $\mathbf{s}_1$. In other words, we have

$$\mathbf{s}_1 \in \text{row}\{\mathbf{Y}_0\mathbf{C}_{1,1}^H\} \cap \text{row}\{\mathbf{Y}_0\mathbf{C}_{1,2}^H\}$$

where row$\{\cdot\}$ represents the row space. This may uniquely determine $\mathbf{s}_1$ (up to a complex scaling factor).

We now discuss the proposed deterministic blind source separation algorithm in detail. For the sake of clarity, let us first assume that no additive noise is present. Calculating the singular value decomposition (SVD) [16] of $\mathbf{Y}_k$ ($k = A_1, A_1 + 1, \cdots, A_2$) leads to

$$\mathbf{Y}_k = \mathbf{U}_k \boldsymbol{\Sigma}_k \mathbf{V}_k^H$$

where $\boldsymbol{\Sigma}_k$ is a diagonal matrix (diagonal elements in descending order) of the same size as $\mathbf{Y}_k$, and $\mathbf{U}_k$ and $\mathbf{V}_k$ are square unitary matrices. Because of Assumptions 1 and 2, $\mathbf{Y}_k$ has rank r, and

$$\text{row}\{\mathbf{X}_k\} = \text{row}\{\mathbf{Y}_k\}.$$

Defining the $K \times r$ matrix $\mathbf{V}_k^s$ as the collection of the first r columns of $\mathbf{V}_k$ and the $K \times (K-r)$ matrix $\mathbf{V}_k^n$ as the collection of the last $K - r$ columns of $\mathbf{V}_k$, we can then write that

$$\text{row}\{\mathbf{X}_k\} = \text{row}\left\{\mathbf{V}_k^{sH}\right\} \Leftrightarrow \text{row}\left\{\mathbf{X}_k\right\} = \left(\text{row}\left\{\mathbf{V}_k^{nH}\right\}\right)^{\perp} \tag{11}$$

where $(\cdot)^{\perp}$ represents the orthogonal complement. Since $\mathbf{x}_{j,p}$ ($p = 1, 2 \cdots, P$) is a row of $\mathbf{X}_k$, we obtain

$$\mathbf{x}_{j,p} \in \text{row}\left\{\mathbf{V}_k^{sH}\right\} \Leftrightarrow \mathbf{x}_{j,p} \in \left(\text{row}\left\{\mathbf{V}_k^{nH}\right\}\right)^{\perp}. \tag{12}$$

Because $\mathbf{x}_{j,p} = \mathbf{s}_j \mathbf{C}_{j,p}$ and $\mathbf{C}_{j,p}\mathbf{C}_{j,p}^H = \mathbf{I}$ [this is due to (2)], (12) can be rewritten as

$$\mathbf{s}_j \in \text{row}\left\{\mathbf{V}_k^{sH}\mathbf{C}_{j,p}^H\right\} \Leftrightarrow \mathbf{s}_j \in \left(\text{row}\left\{\mathbf{V}_k^{nH}\mathbf{C}_{j,p}^H\right\}\right)^{\perp}.$$

This can be derived for every p ($p = 1, 2, \cdots, P$) and for every k ($k = A_1, A_1 + 1, \cdots, A_2$). All these results can then be combined, leading to

$$\mathbf{s}_j \in \bigcap_{k=A_1}^{A_2} \bigcap_{p=1}^{P} \text{row}\left\{\mathbf{V}_k^{sH}\mathbf{C}_{j,p}^H\right\}$$
$$\Leftrightarrow \mathbf{s}_j \in \left(\bigcup_{k=A_1}^{A_2} \bigcup_{p=1}^{P} \text{row}\left\{\mathbf{V}_k^{nH}\mathbf{C}_{j,p}^H\right\}\right)^{\perp}. \tag{13}$$

A vector that satisfies (13) can be found by computing the left singular vector of $\mathbf{C}_j\mathbf{V}^n$ corresponding to the smallest singular value (which is equal to 0) or, equivalently (see [9, Appendix A]), by computing the left singular vector of $\mathbf{C}_j\mathbf{V}^s$ corresponding to the largest singular value (which is equal to $\sqrt{K}$), where $\mathbf{C}_j$ is the $K \times APK$ matrix given by

$$\mathbf{C}_j = [\mathbf{C}_{j,1} \cdots \mathbf{C}_{j,P} | \cdots | \mathbf{C}_{j,1} \cdots \mathbf{C}_{j,P}] \tag{14}$$

$\mathbf{V}^n$ is the $APK \times AP(K-r)$ matrix given by

$$\mathbf{V}^n = \left[\begin{array}{c|c|c} \begin{matrix} \mathbf{V}_{A_1}^n & & \\ & \ddots & \\ & & \mathbf{V}_{A_1}^n \end{matrix} & \mathbf{O} & \mathbf{O} \\ \hline \mathbf{O} & \ddots & \mathbf{O} \\ \hline \mathbf{O} & \mathbf{O} & \begin{matrix} \mathbf{V}_{A_2}^n & & \\ & \ddots & \\ & & \mathbf{V}_{A_2}^n \end{matrix} \end{array}\right] \tag{15}$$

and $\mathbf{V}^s$ is the $APK \times APr$ matrix given by

$$\mathbf{V}^s = \left[\begin{array}{c|c|c} \begin{matrix} \mathbf{V}_{A_1}^s & & \\ & \ddots & \\ & & \mathbf{V}_{A_1}^s \end{matrix} & \mathbf{O} & \mathbf{O} \\ \hline \mathbf{O} & \ddots & \mathbf{O} \\ \hline \mathbf{O} & \mathbf{O} & \begin{matrix} \mathbf{V}_{A_2}^s & & \\ & \ddots & \\ & & \mathbf{V}_{A_2}^s \end{matrix} \end{array}\right]. \tag{16}$$

Let us then introduce the following assumption.

Assumption 3: For any vector $\mathbf{s}'_j$ in $\mathbb{C}^{1\times K}$ linearly independent of $\mathbf{s}_j$, there exists an input matrix $\mathbf{X}_k$ with $A_1 \le k \le A_2$ and a code matrix $\mathbf{C}_{j,p}$ with $1 \le p \le P$ such that

$$\begin{bmatrix} \mathbf{s}'_j\mathbf{C}_{j,p} \\ \mathbf{X}_k \end{bmatrix}$$

has full row rank $r+1$.

Using this assumption, we have the following identifiability result.

Theorem 1: Under Assumptions 1 and 2, we can state that (13) uniquely determines $\mathbf{s}_j$ (up to a complex scaling factor) if and only if Assumption 3 is satisfied.

Proof: Under Assumptions 1 and 2, we know that $\mathbf{s}_j$ satisfies (13). We now prove that Assumption 3 is a necessary and sufficient condition for $\mathbf{s}_j$ to be uniquely determined by (13) (up to a complex scaling factor).

We first prove that Assumption 3 is a necessary condition. Suppose that there exists a vector $\mathbf{s}'_j$ in $\mathbb{C}^{1\times K}$ linearly independent of $\mathbf{s}_j$ such that

$$\begin{bmatrix} \mathbf{s}'_j\mathbf{C}_{j,p} \\ \mathbf{X}_k \end{bmatrix}$$

has a rank lower than $r+1$ for $k = A_1, A_1+1, \cdots, A_2$ and $p = 1, 2, \cdots, P$ (due to Assumption 2, the rank will then actually be r). From (11), it is then clear that

$$\mathbf{s}'_j\mathbf{C}_{j,p} \in \text{row}\left\{\mathbf{V}_k^{sH}\right\} \Leftrightarrow \mathbf{s}'_j \in \text{row}\left\{\mathbf{V}_k^{sH}\mathbf{C}_{j,p}^H\right\}$$
$$\text{for } p = 1, 2, \cdots, P \text{ and } k = A_1, A_1+1, \cdots, A_2.$$

This means that (13) is also satisfied for $\mathbf{s}'_j$.

We then prove that Assumption 3 is a sufficient condition. Suppose that there exists a vector $\mathbf{s}'_j$ in $\mathbb{C}^{1\times K}$ linearly independent of $\mathbf{s}_j$ such that (13) is also satisfied for $\mathbf{s}'_j$. This means that

$$\mathbf{s}'_j \in \text{row}\left\{\mathbf{V}_k^{sH}\mathbf{C}_{j,p}^H\right\} \Leftrightarrow \mathbf{s}'_j\mathbf{C}_{j,p} \in \text{row}\left\{\mathbf{V}_k^{sH}\right\}$$
$$\text{for } p = 1, 2, \cdots, P \text{ and } k = A_1, A_1+1, \cdots, A_2.$$

From (11), it is then clear that

$$\begin{bmatrix} \mathbf{s}'_j\mathbf{C}_{j,p} \\ \mathbf{X}_k \end{bmatrix}$$

has a rank lower than $r+1$ for $k = A_1, A_1+1, \cdots, A_2$ and $p = 1, 2, \cdots, P$ (due to Assumption 2, the rank will then actually be r). This concludes the proof. ∎

Assumption 3 is satisfied if there exists an input matrix $\mathbf{X}_k$ with $A_1 \le k \le A_2$ such that

$$\left[\begin{array}{ccc} \mathbf{C}_{j,1} & \cdots & \mathbf{C}_{j,P} \\ \hline \mathbf{X}_k & & \\ & \ddots & \\ & & \mathbf{X}_k \end{array}\right]$$

has a one-dimensional (1-D) left null space or, equivalently, has rank $K + Pr - 1$. For random complex or real code sequences and random complex or real data symbol sequences, this is the case with probability 1 if

$$K(P-1) \ge Pr - 1$$

which indicates that $P \ge 2$ should be used. To support this claim, it is shown in the next remark that Assumption 3 is most likely not satisfied for $P = 1$.

Remark 2: Let us take $P = 1$ and focus on the first user. If we assume that $L_1 \le L_j$, with $j \ne 1$, we know that $\mathbf{x}_{j,1} = \mathbf{s}_j\mathbf{C}_{j,1}$ is a row of every input matrix from the set $\{\mathbf{X}_k\}_{a=-Q}^{L_1}$. This means that if we take $\mathbf{s}'_1 = \mathbf{s}_j\mathbf{C}_{j,1}\mathbf{C}_{1,1}^{-1}$, $\mathbf{s}'_1\mathbf{C}_{1,1}$ is also a row of every input matrix from the set $\{\mathbf{X}_k\}_{a=-Q}^{L_1}$ (because $\mathbf{s}'_1\mathbf{C}_{1,1} = \mathbf{s}_j\mathbf{C}_{j,1} = \mathbf{x}_{j,1}$). If we further assume that $\mathbf{x}_{j,1}$ is independent from $\mathbf{x}_{1,1}$, we further know that $\mathbf{s}'_1$ is independent from $\mathbf{s}_1$. Hence, Assumption 3 is then not satisfied, irrespective of A_1 and A_2, with $-Q \le A_1 \le A_2 \le L_1$.

Note that robustness against channel order underestimation is obtained by the fact that Assumption 3 can very well be satisfied for $A_2 < L_j$.

Let us then assume additive noise is present. Calculating the SVD of $\mathbf{Y}_k$ ($k = A_1, A_1+1, \cdots, A_2$) then leads to

$$\mathbf{Y}_k = \hat{\mathbf{U}}_k\hat{\mathbf{\Sigma}}_k\hat{\mathbf{V}}_k^H$$

where $\hat{\mathbf{\Sigma}}_k$ is a diagonal matrix (diagonal elements in descending order) of the same size as $\mathbf{Y}_k$ and $\hat{\mathbf{U}}_k$ and $\hat{\mathbf{V}}_k$ are square unitary matrices. For an estimate $\hat{r}$ of the system order r, let us then define the $K \times (K - \hat{r})$ matrix $\hat{\mathbf{V}}_k^n$ as the collection of the last $K - \hat{r}$ columns of $\hat{\mathbf{V}}_k$ and the $K \times \hat{r}$ matrix $\hat{\mathbf{V}}_k^s$ as the collection of the first $\hat{r}$ columns of $\hat{\mathbf{V}}_k$. In correspondence with the noiseless case, we then compute the left singular vector of $\mathbf{C}_j\hat{\mathbf{V}}^n$ corresponding to the smallest singular value (noise-subspace version of the proposed algorithm) or, equivalently (see [9, Appendix A]), we then compute the left singular vector of $\mathbf{C}_j\hat{\mathbf{V}}^s$ corresponding to the largest singular value (signal-subspace version of the proposed algorithm), where $\hat{\mathbf{V}}^n$ is the $APK \times AP(K - \hat{r})$ matrix, which is defined in a similar fashion as $\mathbf{V}^n$ [see (15)] using $\hat{\mathbf{V}}_k^n$ instead of $\mathbf{V}_k^n$, and $\hat{\mathbf{V}}^s$ is the $APK \times AP\hat{r}$ matrix, which is defined in a similar fashion as $\mathbf{V}^s$ [see (16)] using $\hat{\mathbf{V}}_k^s$ instead of $\mathbf{V}_k^s$. Note that if $K - \hat{r} < \hat{r}$, the noise-subspace version is less complex than the signal-subspace version, whereas if $K - \hat{r} > \hat{r}$, it is the other way around. The proposed deterministic blind source separation algorithm is summarized in Table I. The corresponding parameter restrictions are summarized in Table II.

A. Further Discussion

1) The effect of the additive noise on $\hat{\mathbf{V}}_k^n$ and $\hat{\mathbf{V}}_k^s$ can be computed using the first order perturbation analysis [17]. The result can be used to derive a statistically optimal weighting matrix. However, as demonstrated in [18] in a somewhat different context, applying this weighting matrix should be avoided.
2) When we take $\hat{r}$ equal to the number of rows of $\mathbf{Y}_k$ ($\hat{r} = (Q+1)M$), we can calculate $\hat{\mathbf{V}}_k^n$ or $\hat{\mathbf{V}}_k^s$ from a QR decomposition (QRD) [16] of $\mathbf{Y}_k$. This results in a significant complexity reduction.
3) Following a similar approach as in [19], where a single-user system without coding is considered, and [20], where a multiuser DS-CDMA system is considered, it is also possible to derive a direct blind equalizer

TABLE I
DETERMINISTIC BLIND SOURCE SEPARATION ALGORITHM

1. for $k = A_1, A_1+1, \ldots, A_2$:
   - compute the SVD of $\mathbf{Y}_k$: $\mathbf{Y}_k = \hat{\mathbf{U}}_k \hat{\mathbf{\Sigma}}_k \hat{\mathbf{V}}_k^H$
   - estimate the system order r: $\hat{r}$
   - if $K - \hat{r} \leq \hat{r}$:
     - ∗ collect the last $K - \hat{r}$ columns of $\hat{\mathbf{V}}_k$: $\hat{\mathbf{V}}_k^n$

     else:
     - ∗ collect the first $\hat{r}$ columns of $\hat{\mathbf{V}}_k$: $\hat{\mathbf{V}}_k^s$
2. if $K - \hat{r} \leq \hat{r}$:
   - construct $\hat{\mathbf{V}}^n$ (see (15)) and $\mathbf{C}_j$ (see (14))
   - solve $\min_{\mathbf{s}_j}\{\|\mathbf{s}_j \mathbf{C}_j \hat{\mathbf{V}}^n\|^2\}$, s.t. $\|\mathbf{s}_j\|^2 = 1$

   else:
   - construct $\hat{\mathbf{V}}^s$ (see (16)) and $\mathbf{C}_j$ (see (14))
   - solve $\max_{\mathbf{s}_j}\{\|\mathbf{s}_j \mathbf{C}_j \hat{\mathbf{V}}^s\|^2\}$, s.t. $\|\mathbf{s}_j\|^2 = 1$

TABLE II
PARAMETER RESTRICTIONS

1. $-Q \leq A_1 \leq A_2 \leq L_j$
2. $(Q+1)M \geq r = (Q+1)JP + P\sum_{j=1}^{J} L_j \Rightarrow M > JP$
3. $K \geq r = (Q+1)JP + P\sum_{j=1}^{J} L_j$
4. $K(P-1) \geq Pr - 1 \Rightarrow P \geq 2$

estimation algorithm that is related to the proposed direct blind symbol estimation algorithm.

4) Instead of working with the SVDs [or QRDs if $\hat{r} = (Q+1)M$] of the $(Q+1)M \times K$ output matrices from the set $\{\mathbf{Y}_k\}_{k=A_1}^{A_2}$, we could also follow the approach presented in [9] and [10] and work with the SVD [or QRD if $\hat{r} = (Q+1)M$] of the $(Q+1)M \times (K - Q - \hat{L}_{\min})$ output matrix

$$\begin{bmatrix} \mathbf{y}[\hat{L}_{\min}] & \mathbf{y}[\hat{L}_{\min}+1] & \cdots & \mathbf{y}[K-Q-1] \\ \vdots & \vdots & & \vdots \\ \mathbf{y}[\hat{L}_{\min}+Q] & \mathbf{y}[\hat{L}_{\min}+Q+1] & \cdots & \mathbf{y}[K-1] \end{bmatrix}$$

where $\hat{L}_{\min}$ is an estimate of the minimal channel order $L_{\min}$ smaller than or equal to $L_{\min}$ ($\hat{L}_{\min} \leq L_{\min}$). However, when calculating the SVD [or QRD if $\hat{r} = (Q+1)M$] of one output matrix from the set $\{\mathbf{Y}_k\}_{k=A_1}^{A_2}$ and calculating the SVDs [or QRDs if $\hat{r} = (Q+1)M$] of the other output matrices from that set using an adaptive SVD algorithm [or adaptive QRD algorithm if $\hat{r} = (Q+1)M$], there is not much difference in complexity between the approach we follow and the approach presented in [9] and [10]. Moreover, the approach we follow lends itself better to a possible adaptive implementation (see the next point).

5) The noise-subspace version of the proposed algorithm can also be implemented in an adaptive way using an RLS scheme. We therefore refer to [20], where a similar problem is discussed in the context of a multiuser DS-CDMA system. To exploit the finite alphabet property of the data symbols, [20] also describes a Viterbi algorithm, which can easily be adapted for the multiuser system under consideration. Note that an interesting Viterbi algorithm for a multiuser system employing linear block coding is introduced in [21] (see also [22]).

B. Modifications for a Real Constellation

When the data symbols belong to a real constellation, the realness of the constellation can be exploited. When no coding is used, this is usually done by splitting the received sequence in its real and imaginary part, hence doubling the number of observations prior to any other operation (see [1], [4], [9], and [23]). Here, we use a somewhat different approach.

When the data symbols belong to a real constellation and we assume no additive noise is present, we can rewrite (13) as

$$\mathbf{s}_j \in \bigcap_{k=A_1}^{A_2} \bigcap_{p=1}^{P} \text{row}\left\{ \begin{bmatrix} \Re\{\mathbf{V}_k^s\} & \Im\{\mathbf{V}_k^s\} \\ -\Im\{\mathbf{V}_k^s\} & \Re\{\mathbf{V}_k^s\} \end{bmatrix}^T \cdot [\Re\{\mathbf{C}_{j,p}\} \quad \Im\{\mathbf{C}_{j,p}\}]^T \right\}$$
$$\Updownarrow$$
$$\mathbf{s}_j \in \left(\bigcup_{k=A_1}^{A_2} \bigcup_{p=1}^{P} \text{row}\left\{ \begin{bmatrix} \Re\{\mathbf{V}_k^n\} & \Im\{\mathbf{V}_k^n\} \\ -\Im\{\mathbf{V}_k^n\} & \Re\{\mathbf{V}_k^n\} \end{bmatrix}^T \cdot [\Re\{\mathbf{C}_{j,p}\} \quad \Im\{\mathbf{C}_{j,p}\}]^T \right\} \right)^{\perp}. \tag{17}$$

A vector that satisfies (17) can be found by computing the left singular vector of

$$[\Re\{\mathbf{C}_j\} \quad \Im\{\mathbf{C}_j\}] \begin{bmatrix} \Re\{\mathbf{V}^n\} & \Im\{\mathbf{V}^n\} \\ -\Im\{\mathbf{V}^n\} & \Re\{\mathbf{V}^n\} \end{bmatrix}$$

corresponding to the smallest singular value (which is equal to 0) or, equivalently (see [9, App. A]), by computing the left singular vector of

$$[\Re\{\mathbf{C}_j\} \quad \Im\{\mathbf{C}_j\}] \begin{bmatrix} \Re\{\mathbf{V}^s\} & \Im\{\mathbf{V}^s\} \\ -\Im\{\mathbf{V}^s\} & \Re\{\mathbf{V}^s\} \end{bmatrix}$$

corresponding to the largest singular value (which is equal to $\sqrt{K}$).

Let us then introduce the following assumption.

Assumption 4: For any vector $\mathbf{s}'_j$ in $\mathbb{R}^{1\times K}$ linearly independent of $\mathbf{s}_j$, there exists an input matrix $\mathbf{X}_k$ with $A_1 \leq k \leq A_2$ and a code matrix $\mathbf{C}_{j,p}$ with $1 \leq p \leq P$ such that

$$\begin{bmatrix} \mathbf{s}'_j \Re\{\mathbf{C}_{j,p}\} & \mathbf{s}'_j \Im\{\mathbf{C}_{j,p}\} \\ \Re\{\mathbf{X}_k\} & \Im\{\mathbf{X}_k\} \\ -\Im\{\mathbf{X}_k\} & \Re\{\mathbf{X}_k\} \end{bmatrix}$$

has full row rank $2r+1$.

Using this assumption, we have the following identifiability result.

Theorem 2: Under Assumptions 1 and 2, we can state that (17) uniquely determines $\mathbf{s}_j$ (up to a real scaling factor) if and only if Assumption 4 is satisfied.

Proof: The proof is similar as the proof of Theorem 1. ■

Assumption 4 is satisfied if there exists an input matrix $\mathbf{X}_k$ with $A_1 \leq k \leq A_2$ such that

$$\left[\begin{array}{ccc|ccc} \Re\{\mathbf{C}_{j,1}\} & \cdots & \Re\{\mathbf{C}_{j,P}\} & \Im\{\mathbf{C}_{j,1}\} & \cdots & \Im\{\mathbf{C}_{j,P}\} \\ \hline \Re\{\mathbf{X}_k\} & & & \Im\{\mathbf{X}_k\} & & \\ & \ddots & & & \ddots & \\ & & \Re\{\mathbf{X}_k\} & & & \Im\{\mathbf{X}_k\} \\ \hline -\Im\{\mathbf{X}_k\} & & & \Re\{\mathbf{X}_k\} & & \\ & \ddots & & & \ddots & \\ & & -\Im\{\mathbf{X}_k\} & & & \Re\{\mathbf{X}_k\} \end{array}\right]$$

has a 1-D left null space or, equivalently, has rank $K + 2Pr - 1$. For random complex code sequences and random real data symbol sequences, this is the case with probability 1 if

$$K(2P - 1) \geq 2Pr - 1$$

which indicates that any $P \geq 1$ can now be used.

Note that robustness against channel order underestimation is obtained by the fact that Assumption 4 can very well be satisfied for $A_2 < L_j$.

V. Simulation Results

We assume that the data symbol sequences $\{s_j[k]\}_{j=1}^{J}$ are mutually independent and zero-mean white with variance 1. We further assume that the additive noises $\{e^{(m)}[n]\}_{m=1}^{M}$ are mutually independent and zero-mean white Gaussian with variance σ_e^2. For simplicity, we also assume that

$$\sum_{k=0}^{L_j} \|\mathbf{g}_{j,p}[k]\|^2 = 1, \qquad \text{for } j = 1, 2, \cdots, J \quad \text{and } p = 1, 2, \cdots, P.$$

Using (2), the signal-to-noise ratio (SNR) for every user at the input of the receiver can then be defined as

$$\text{SNR} = \frac{P}{M\sigma_e^2}.$$

For all simulations, we will conduct 2000 trials, using bursts of $K = 100$ data symbols.

A. Convolutive Single-User System

In this subsection, we perform some simulations on a convolutive single-user system ($J = 1$ and $L_1 = 4$). We consider BPSK modulation, $M = 4$ receive antennas, and $P = 1$ transmit antenna and take $Q = 1$. We examine two scenarios.

1) $A_1 = -Q = -1$ and $A_2 = L_1 = 4$.
2) $A_1 = -Q = -1$ and $A_2 = 0$.

The condition number of $\mathcal{G}$ is 6.8907.

We first assume that $c_{1,1}[k]$ is a random complex code sequence and apply the proposed algorithm. To exploit the realness of the constellation, we use the modifications discussed in Section IV-B. We only consider $\hat{r} = r = 6$.

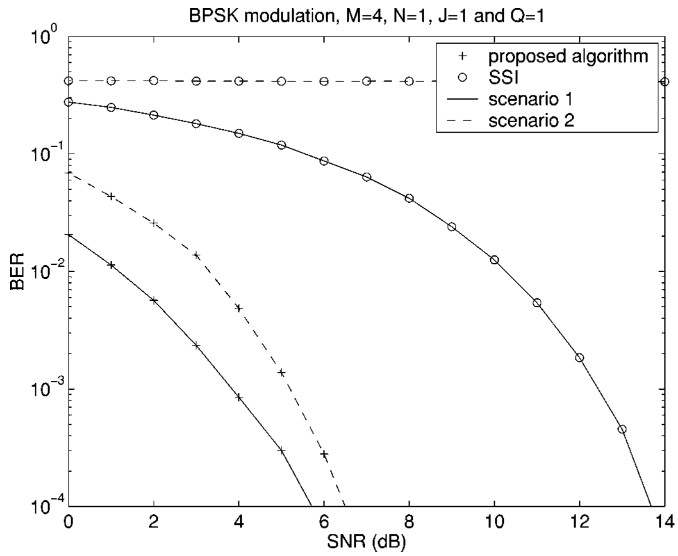

Fig. 2. BER as a function of the SNR for two different algorithms (convolutive single-user system, BPSK modulation, one transmit antenna).

We next assume that $c_{1,1}[k] = 1$ (no coding) and apply the SSI algorithm presented in [19] (note that this SSI algorithm is slightly different from the SSI algorithms presented in [9] and [10]). To exploit the realness of the constellation, we split the received sequence in its real and imaginary part. We only consider $\hat{r} = r = 6$.

Note that considering $\hat{r} = r = 6$ actually means that we know that $L_1 = 4$ (since we take $Q = 1$). Hence, scenario 2 maybe seems somewhat artificial. However, the conclusions we draw from the simulations (see next paragraph) also hold when we consider $\hat{r} > r = 6$, in which case, scenario 2 does make sense.

Fig. 2 shows the BER as a function of the SNR for the two algorithms. First of all, we see that if we use the correct channel order, the performance of the proposed algorithm is much better than the performance of the SSI algorithm presented in [19]. Next, we observe that if we underestimate the channel order, the proposed algorithm still works, whereas the SSI algorithm presented in [19] does not.

B. Instantaneous Mixture

In this subsection, we perform some simulations on an instantaneous mixture ($J = 4$ and $L_1 = L_2 = L_3 = L_4 = 0$). We consider BPSK modulation, $M = 6$ receive antennas, and $P = 1$ transmit antenna per user and take $Q = 0$ (temporal smoothing has no use for an instantaneous mixture). Hence, we can only examine $A_1 = A_2 = 0$ for every user. The condition number of $\mathcal{G}$ is 2.9238.

We first assume that $c_{1,1}[k]$, $c_{2,1}[k]$, $c_{3,1}[k]$, and $c_{4,1}[k]$ are random complex code sequences and apply the proposed algorithm. To exploit the realness of the constellation, we use the modifications discussed in Section IV-B. We only consider $\hat{r} = r = 4$.

We next assume that $c_{1,1}[k] = c_{2,1}[k] = c_{3,1}[k] = c_{4,1}[k] = 1$ (no coding) and apply the ILSP algorithm [1] and the RACMA algorithm [4]. To exploit the realness of the constellation, we split the received sequence in its real and imaginary part. For the ILSP algorithm, we consider different numbers of random

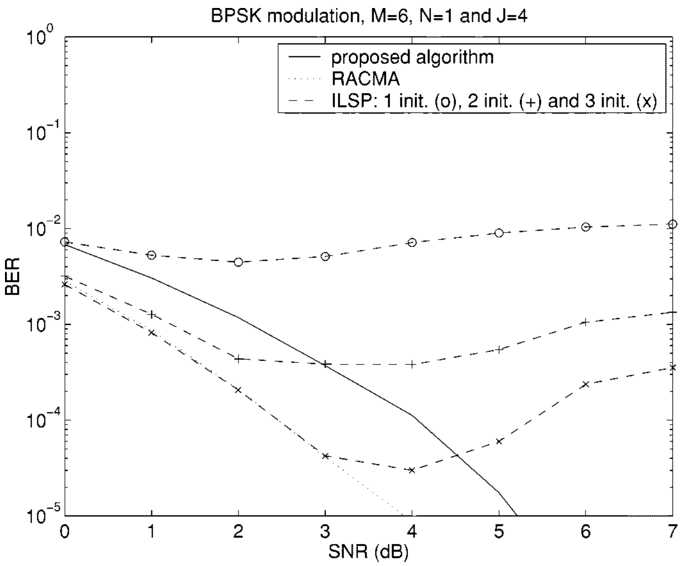

Fig. 3. Average BER per user as a function of the SNR for three different algorithms (instantaneous mixture, BPSK modulation, one transmit antenna per user).

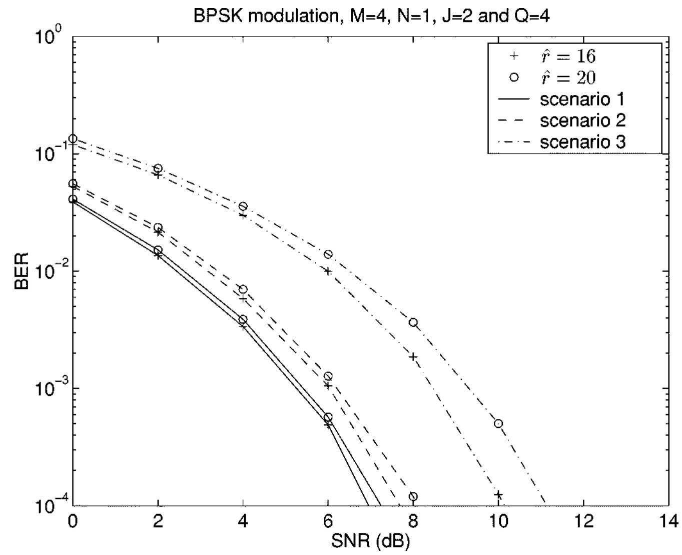

Fig. 4. Average BER per user as a function of the SNR (convolutive mixture, BPSK modulation, one transmit antenna per user).

initializations (one, two, and three random initializations). For the RACMA algorithm, we only consider $\hat{r} = r = 4$.

Fig. 3 shows the average BER per user as a function of the SNR for the three algorithms. We observe that the performance of the ILSP algorithm strongly depends on the number of random initializations. We also see that for a small number of random initializations and a high SNR, the ILSP algorithm may not find the global minimum. The good performance of the ILSP algorithm (for a large number of random initializations and a low SNR) and the RACMA algorithm can be explained by the fact that these two algorithms jointly detect all transmitted data symbol sequences and that they exploit the finite alphabet property of the data symbols. Although the proposed algorithm does not have these properties, its performance is fairly close to the performance of the ILSP algorithm (for a large number of random initializations and a low SNR) and the RACMA algorithm.

C. Convolutive Mixture

Finally, we perform some simulations on a convolutive mixture ($J = 2$, $L_1 = 4$ and $L_2 = 2$).

We first consider BPSK modulation, $M = 4$ receive antennas, and $P = 1$ transmit antenna per user and take $Q = 4$. We examine three scenarios.

1) $A_1 = -Q = -4$ and $A_2 = L_1 = 4$ for the first user and $A_1 = -Q = -4$ and $A_2 = L_2 = 2$ for the second user.
2) $A_1 = -Q = -4$ and $A_2 = L_{\min} = 2$ for every user.
3) $A_1 = -Q = -4$ and $A_2 = 0$ for every user.

The condition number of $\mathcal{G}$ is 12.6795. We assume that $c_{1,1}[k]$ and $c_{2,1}[k]$ are random complex code sequences and apply the proposed algorithm. To exploit the realness of the constellation, we use the modifications discussed in Section IV-B. We consider two values of $\hat{r}$.

1) $\hat{r} = r = 16$.
2) $\hat{r} = (Q+1)M = 20$.

Fig. 4 shows the average BER per user as a function of the SNR for this setup.

We next consider QPSK modulation, $M = 8$ receive antennas, and $P = 2$ transmit antennas per user and take $Q = 4$. We examine three scenarios.

1) $A_1 = -Q = -4$ and $A_2 = L_1 = 4$ for the first user and $A_1 = -Q = -4$ and $A_2 = L_2 = 2$ for the second user.
2) $A_1 = -Q = -4$ and $A_2 = L_{\min} = 2$ for every user.
3) $A_1 = -Q = -4$ and $A_2 = 0$ for every user.

The condition number of $\mathcal{G}$ is 15.1524. We assume that $c_{1,1}[k]$, $c_{2,1}[k]$, $c_{1,2}[k]$, and $c_{2,2}[k]$ are random complex code sequences and apply the proposed algorithm. We consider two values of $\hat{r}$.

1) $\hat{r} = r = 32$.
2) $\hat{r} = (Q+1)M = 40$.

Fig. 5 shows the average BER per user as a function of the SNR for this setup.

We again observe that the proposed algorithm is robust against channel order underestimation. Moreover, we see that it is also fairly robust against system order overestimation.

VI. Conclusions

We have presented a new simple deterministic blind source separation algorithm, which is based on modulating the same data symbol sequence with different code sequences and transmitting the resulting modulated data symbol sequences through different antennas. The algorithm does not exploit the finite alphabet property of the data symbols. As a result, no iterations are required, and convergence is not an issue. Instantaneous mixtures (frequency-flat fading), as well as convolutive mixtures (frequency-selective fading), can be handled. In the case of a convolutive mixture, the difficulties that occur when the users have unequal channel orders are avoided. Moreover, the proposed algorithm is robust against channel order underestimation.

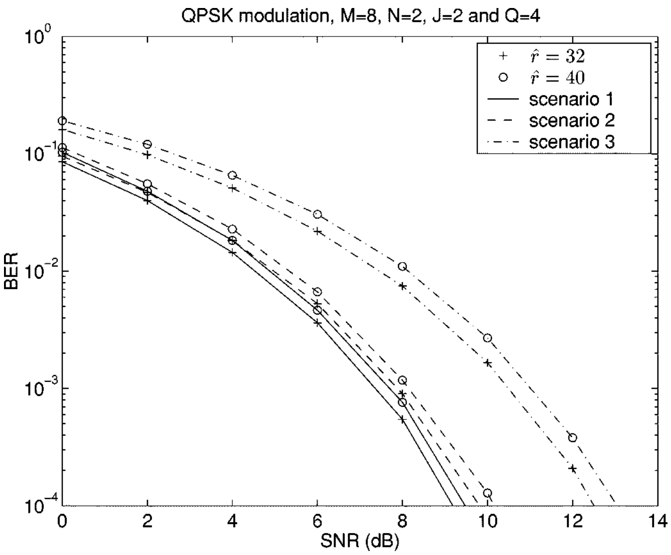

Fig. 5. Average BER per user as a function of the SNR (convolutive mixture, QPSK modulation, two transmit antennas per user).

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Geert Leus was born in Leuven, Belgium, in 1973. He received the electrical engineering degree and the Ph.D. degree in applied sciences from the Katholieke Universiteit Leuven (KU Leuven) in 1996 and 2000, respectively.

Currently, he is a Postdoctoral Fellow of the Fund for Scientific Research, Flanders (FWO, Vlaanderen) at the Electrical Engineering Department, KU Leuven. His research interests are in the area of signal processing for digital communications.

Piet Vandaele was born in Diksmuide, Belgium, in 1972. He received the electrical engineering degree and the Ph.D. degree in applied sciences from the Katholieke Universiteit Leuven, Leuven, Belgium, in 1995 and 1999, respectively.

In 2000, he joined the Corporate Research Center of Alcatel, Antwerp, Belgium, where he is working on multicarrier systems for xDSL transmission. His research interests are in the area of signal processing for digital communications.

Dr. Vandaele received the Alcatel Bell Award (with M. Moonen) in 1997.

Marc Moonen was born in St.-Truiden, Belgium, in 1963. He received the electrical engineering degree and the Ph.D. degree in applied sciences from the Katholieke Universiteit Leuven (KU Leuven), Leuven, Belgium, in 1986 and 1990, respectively.

Since 1994, he has been a Research Associate with the Belgian NFWO (National Fund for Scientific Research) at the Electrical Engineering Department, KU Leuven. His research activities are in mathematical systems theory and signal processing, parallel computing, and digital communications. He is a member of the editorial board of *Integration, the VLSI Journal*.

Dr. Moonen received the 1994 KU Leuven Research Council Award, the 1997 Alcatel Bell (Belgium) Award (with P. Vandaele), and was a 1997 “Laureate of the Belgium Royal Academy of Science.” He is Chairman of the IEEE Benelux Signal Processing Chapter, a EURASIP Officer.