

# RLS-Based Initialization for Per-Tone Equalizers in DMT Receivers

Katleen Van Acker, Geert Leus, Marc Moonen, and Thierry Pollet

**Abstract**—Per-tone equalization has recently been proposed as an alternative receiver structure for discrete multitone-based systems improving upon the well-known structure based on time-domain equalization. Fast initialization of all the equalizer coefficients has been identified as an open problem. In this letter, a recursive initialization scheme based on recursive least squares with inverse updating is presented for the per-tone equalizers. Simulation results show convergence with an acceptably small number of training symbols. Complexity calculations are made for per-tone equalization and for the case where tones are grouped. It is demonstrated with an example that in the latter case, initialization complexity becomes sufficiently low and comparable to complexity during data transmission.

**Index Terms**—Asymmetric digital subscriber line (ADSL), discrete multitone (DMT), equalization, recursive least squares (RLS).

## I. INTRODUCTION

DISCRETE MULTITONE (DMT) has become an important transmission method, for instance, for asymmetric digital subscriber lines (ADSLs) [1]. A conventional equalization scheme for such a DMT modem consists of a time-domain equalizer (TEQ) which shortens the channel impulse response such that the total impulse response is shorter than the cyclic prefix, followed by a one-taps frequency-domain equalizer for each tone [3], [5], [8].

As an alternative to TEQ, *per-tone equalization* has been proposed in [6]. There is no TEQ involved in the system, but a  $T$ -taps per-tone equalizer (PTEQ) is inserted for each tone separately. This scheme enables performing true signal-to-noise ratio (SNR)-optimization per tone, in contrast with the TEQ-based scheme, while complexity during data transmission is kept at the same level. Moreover, PTEQ has been shown to have a significantly reduced sensitivity to the so-called synchronization delay. This delay determines which received samples belong to the same received symbol.

Paper approved by M. Chiani, the Editor for Wireless Communication of the IEEE Communications Society. Manuscript received September 20, 2000; revised July 15, 2002. The work of M. Moonen and G. Leus was supported by the Fund for Scientific Research, Flanders (FWO). This work was supported in part by the Belgian Prime Minister's Office, Federal Office for Scientific, Technical and Cultural Affairs, under IUAP P4-02 (1997–2001): Modeling, Identification, Simulation and Control of Complex Systems; in part by the Concerted Research Action GOA-MEFISTO-666 of the Flemish Government; in part by the Flemish Institute for Scientific and Technological Research in Industry (IWT) [IRMUT (980271) and Advanced Internet Access (980316)]; and in part by Alcatel. This paper was presented in part at the European Signal Processing Conference, Tampere, Finland, September 2000.

K. Van Acker and T. Pollet are with Research and Innovation, Alcatel Bell, 2018 Antwerpen, Belgium (e-mail: katleen.van\_acker@alcatel.be; thierry.pollet@alcatel.be).

G. Leus and M. Moonen are with the Katholieke Universiteit Leuven, ESAT-SISTA, 3001 Leuven, Belgium (e-mail: geert.leus@esat.kuleuven.ac.be; marc.moonen@esat.kuleuven.ac.be).

Digital Object Identifier 10.1109/TCOMM.2003.813176

An initialization formula has been derived in [6] which is applicable when a channel model is available, as well as signal and noise covariance matrices. This direct initialization is computationally intensive. Hence, there is a strong need for a less complex initialization algorithm.

In this letter, a recursive initialization scheme based on so-called recursive least squares (RLS) with inverse updating is presented. It is demonstrated that this scheme achieves initialization with an acceptably small number of training symbols. As a significant part of the RLS computations can be “shared” among the different tones, this is achieved at an acceptable computational cost.

## II. PTEQ IN A DMT MODEM

The concepts of PTEQ are briefly reviewed. For more details, we refer readers to [6], where it is also shown that such an approach clearly outperforms the well-known TEQ (channel shortening) approach.

The per-tone approach is based on transferring the TEQ operations to the frequency domain (i.e., after the fast Fourier transform (FFT) demodulation) which results in a  $T$ -taps PTEQ for each tone separately, fed with the outputs of a sliding FFT operation. At first sight, multiple FFTs are needed for each symbol. But it is demonstrated in [6] that, for every  $T$ -taps PTEQ, there exists a modified  $T$ -taps PTEQ which has as its inputs the corresponding output of only one FFT and  $T-1$  real difference terms. The modified equalizers then have two functions: they equalize the channel impulse response, and at the same time, incorporate the sliding Fourier transform computations, whereas the original equalizers only equalize the channel impulse response. Let us describe this in more detail. Suppose  $N$  is the FFT size and  $T$  is the length of the PTEQ. Collecting the  $N+T-1$  received samples related to symbol period  $k$  in the  $(N+T-1) \times 1$  vector  $\mathbf{y}^{(k)}$ , the PTEQ for the  $i$ th tone, denoted by the  $T \times 1$  vector  $\mathbf{v}_i$ , then has as its inputs the  $i$ th output of the FFT, denoted by  $[\mathbf{0}_{1 \times (T-1)}, \mathcal{F}_N(i, :)] \cdot \mathbf{y}^{(k)}$ , and  $T-1$  difference terms, denoted by  $[\mathbf{I}_{T-1}, \mathbf{0}_{(T-1) \times (N-T+1)}, -\mathbf{I}_{T-1}] \cdot \mathbf{y}^{(k)}$ , where  $\mathbf{I}_n$  is the  $n \times n$  identity matrix,  $\mathbf{0}_{n \times m}$  is the  $n \times m$  all-zero matrix, and  $\mathcal{F}_N(i, :)$  is the  $i$ th row of the  $N \times N$  FFT matrix  $\mathcal{F}_N$ . The optimal modified equalizers, which give rise to maximum SNR on each tone  $i$ , are then found by

$$\begin{aligned} & \min_{\mathbf{v}_i} J(\mathbf{v}_i) \\ & = \min_{\mathbf{v}_i} \mathcal{E} \left\{ \left| \frac{1}{\sqrt{v_i}} \cdot \underbrace{\begin{bmatrix} \mathbf{I}_{T-1} & \mathbf{0} & -\mathbf{I}_{T-1} \\ \mathbf{0} & \mathcal{F}_N(i, :) \end{bmatrix}}_{\mathbf{F}_i} \cdot \mathbf{y}^{(k)} - X_i^{(k)} \right|^2 \right\} \end{aligned} \quad (1)$$

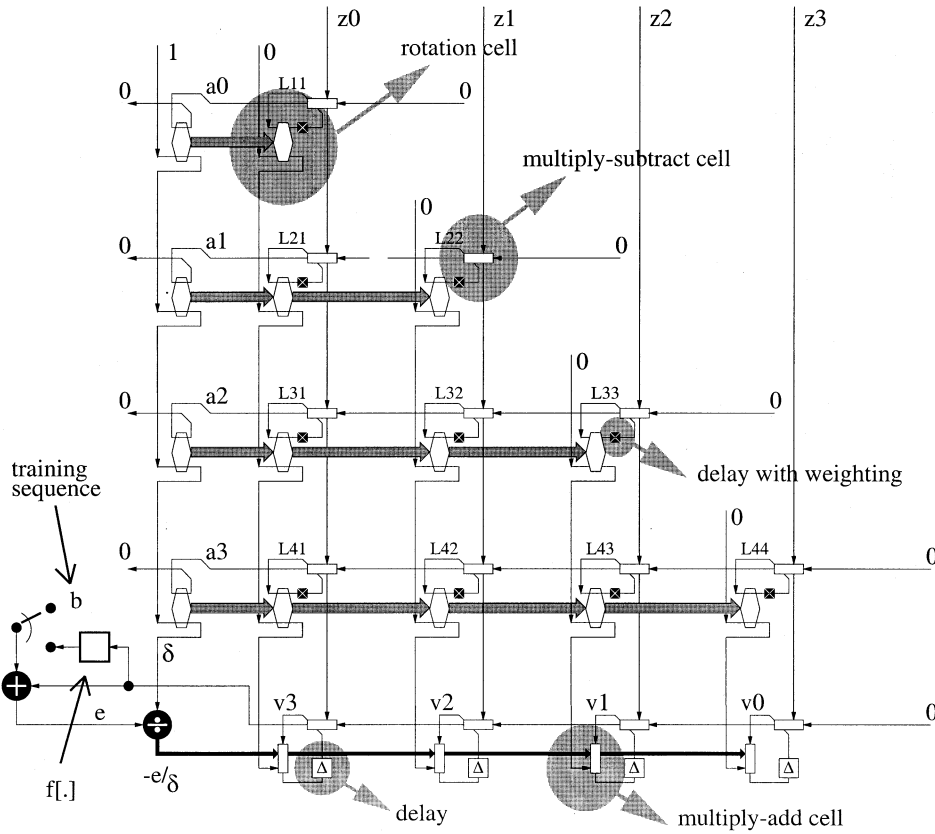


Fig. 1. SFG for inverse updating RLS-based equalization.

where  $\bar{\mathbf{v}}_i$  is obtained by reversing the entries of  $\mathbf{v}_i$ , and  $X_i^{(k)}$  is the subsymbol on tone  $i$  at symbol period  $k$ .

### III. RLS-BASED INITIALIZATION

Direct equalizer coefficient computation, based on the knowledge of the channel impulse response as well as the signal and noise characteristics, has an excessively high computational cost. However, (1) may also be used for a training sequence-based initialization of the PTEQs. If a training sequence  $X_1^{(k)}, \dots, X_N^{(k)}$ ,  $k = 1, \dots, K$ , is transmitted, the optimal  $\mathbf{v}_i$  is computed as [compare to (1)]

$$\min_{\mathbf{v}_i} J_{LS}(\mathbf{v}_i) = \min_{\mathbf{v}_i} \sum_{k=1}^K \left| \bar{\mathbf{v}}_i^T \cdot \mathbf{z}_i^{(k)} - X_i^{(k)} \right|^2 \quad (2)$$

with  $\mathbf{z}_i^{(k)} = \mathbf{F}_i \cdot \mathbf{y}^{(k)}$ . This least-squares estimation problem may be solved recursively.

Least mean squares (LMS)-based schemes [2] for the PTEQs have a low computational complexity. However, they have poor convergence properties and have been shown to require an excessively large number of training symbols.

Here, an initialization based on an RLS scheme [2] for each tone is presented. Such a scheme has optimal convergence properties, and hence, achieves initialization with an acceptably small number of training symbols. More specifically, we use RLS with so-called inverse updating [4]. This scheme is based on storing and updating a lower triangular matrix  $\mathbf{L}_i^{(k)}$ , which is such that  $\mathbf{L}_i^{(k)H} \cdot \mathbf{L}_i^{(k)} = \mathbf{Z}_i^{(k)-1}$ , with  $\mathbf{Z}_i^{(k)}$  the covariance

matrix of the filter input  $\mathbf{Z}_i^{(k)} = \sum_{j=1}^k \mathbf{z}_i^{(j)*} \cdot \mathbf{z}_i^{(j)T}$ , together with the least-squares estimate  $\bar{\mathbf{v}}_i^{(k)}$  at iteration  $k$ . The following formulas, with the tone index  $i$  omitted for compact notation, describe the RLS algorithm.

#### Algorithm Inverse Updating

For  $k = 1 \dots K$

Given:  $\mathbf{L}^{(k-1)}$ , new observation vector  $\mathbf{z}^{(k)}$ , desired response  $X^{(k)}$

Step 1. Form the matrix-vector product

$$\mathbf{a} = -\mathbf{L}^{(k-1)} \cdot \mathbf{z}^{(k)*}.$$

Step 2. For  $j = 1, \dots, T$  determine unitary transformations  $\mathbf{Q}_j$  so that

$$\begin{bmatrix} \mathbf{0}_{(T-1) \times 1} \\ \delta^* \end{bmatrix} \Leftarrow \mathbf{Q}_T \mathbf{Q}_{T-1} \dots \mathbf{Q}_1 \cdot \begin{bmatrix} \mathbf{a} \\ 1 \end{bmatrix}.$$

Step 3. Update  $\mathbf{L}^{(k)}$

$$\begin{bmatrix} \mathbf{L}^{(k)} \\ -\delta^* \cdot \mathbf{k}^{(k)T} \end{bmatrix} \Leftarrow \mathbf{Q}_T \mathbf{Q}_{T-1} \dots \mathbf{Q}_1 \cdot \begin{bmatrix} \mathbf{L}^{(k-1)} \\ \mathbf{0}_{1 \times (T-1)} \end{bmatrix}.$$

Step 4. Update  $\bar{\mathbf{v}}^{(k)}$

$$\bar{\mathbf{v}}^{(k)} = \bar{\mathbf{v}}^{(k-1)} + \left[ -\frac{X^{(k)} - \mathbf{z}^{(k)T} \cdot \bar{\mathbf{v}}^{(k-1)}}{\delta} \right] \cdot \left[ -\delta^* \cdot \mathbf{k}^{(k)T} \right]^H$$

end

$$\bar{\mathbf{v}} = \bar{\mathbf{v}}^{(K)}.$$

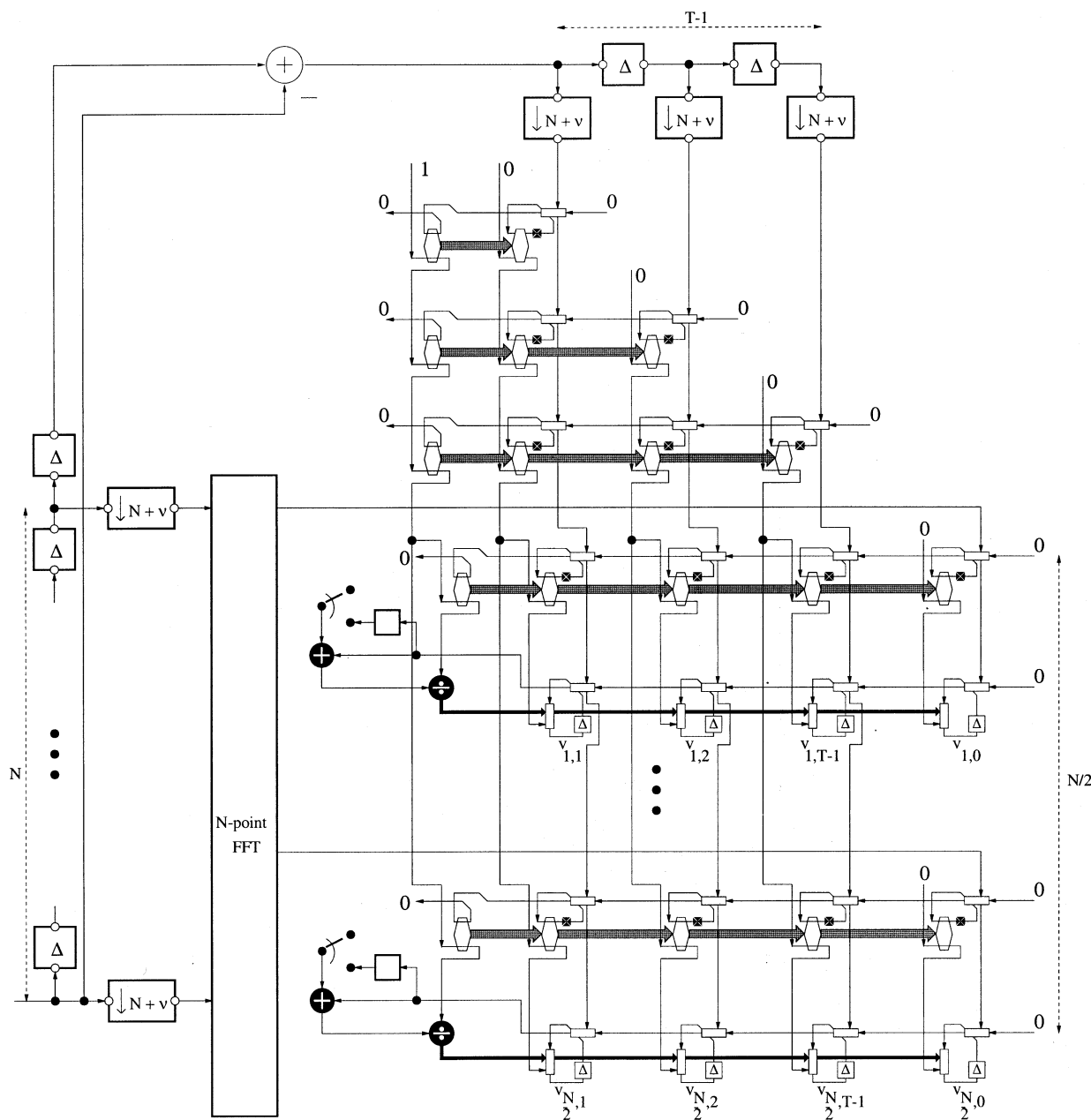


Fig. 2. SFG for inverse updating RLS-based PTEQ.

In *Step 2*,  $Q_j$  performs a complex Givens rotation<sup>1</sup> acting upon the  $j$ th and the last component of

$$Q_{j-1} \cdots Q_1 \cdot \begin{bmatrix} \mathbf{a} \\ 1 \end{bmatrix}$$

such that the  $j$ th component is zeroed. For further details, we refer to [4]. Fig. 1 gives the signal flow graph (SFG) for the RLS algorithm with inverse updating. Note that  $f[\cdot]$  is a decision device. The output of this device may be used as the so-called desired response input, for further training in decision-directed mode, during data transmission.

By applying this algorithm to PTEQ, one obtains Fig. 2. An important aspect in view of the overall computational complexity, is that the  $T - 1$  common real inputs (the difference terms) give rise to a real triangular part in the SFG which is common to all the used tones, i.e., matrix<sup>2</sup>  $\mathbf{L}^{(k)}(1 : T - 1, 1 : T - 1)$ . The complex FFT output is taken as the  $T$ th input to the adaptive filter. This input makes the row  $\mathbf{L}^{(k)}(T, :)$  complex, and different for different tones. Also note that the filter coefficients are updated in the following order:  $[v_{i,1} \cdots v_{i,T-1} \ v_{i,0}]$ , where  $v_{i,0}$ , the tap with complex input, is the right-most coefficient in the SFG. The update of the filter coefficients is also executed separately for each tone. In conclusion, the overall structure contains one real triangular

<sup>1</sup>Represented by a hexagon in the SFG.

<sup>2</sup>Matlab notation.

part, the output of which is used for further processing for each tone separately. For each tone, additional computations are executed corresponding to the last two "rows" in Fig. 1.

#### IV. COMPLEXITY

The complexity numbers given in the sequel are measured in number of *real multiplications*.<sup>3</sup> Initialization complexity of PTEQ based on an RLS scheme for each tone then becomes:

- 1) For the real part of  $\mathbf{L}$  (common for all used tones):
  - a)  $(T - 1)T/2$  for the matrix vector product;
  - b)  $4(((T - 1)T/2) + T - 1)$  for the real rotations (four multiplications per rotation).
- 2) For the complex part of  $\mathbf{L}$  and filter update (per tone):
  - a)  $(T - 1)2 + 4$  for the matrix vector product;
  - b)  $9(T + 1)$  for the complex rotations (nine multiplications per rotation with one real and one complex input);
  - c)  $(T - 1)2 + 4 + 4 + T4$  for the filter update.

The real part is quadratic in  $T$  but shared by all used tones, while the complex part is linear in  $T$  and executed for each tone separately. The latter part will mostly dominate the total complexity, e.g., for ADSL downstream transmission, where  $N_u \gg T$ , with  $N_u$  the total number of used tones. By assuming  $N_u = N/2$  and the symbol duration  $N/F_s$ , we have as total complexity  $8.5F_s \cdot (T + 1) + (F_s/N)(1/2)(5T^2 + 3T - 8)$ .

While performance is almost the same for both per tone and per  $M$  tones equalization if  $M$  is not too large (see [6] and [7] for more details), it can be shown that the overall complexity drops significantly when tones are grouped. The first term reduces to  $(8.5/M)F_s \cdot (T + 1)$  when tones are combined into groups of  $M$  tones. As an example, we take  $N = 512$  (ADSL downstream) and  $M = 11$  (see Section V), which leads to a complexity of roughly  $0.8 \cdot F_s \cdot T + 0.005 \cdot F_s \cdot T^2$ . It is seen that, in this case, initialization complexity is comparable to complexity during data transmission, namely  $F_s \cdot T$  [6].

#### V. SIMULATION RESULTS

Fig. 3 gives some ADSL simulation results for the downstream standard channel T1.601-#9 with additive white noise of  $-140$  dBm/Hz. The symbol size  $N$  is 512, the prefix length is 32, and the used tones are from tone 38 up to tone 256 with power spectral density equal to  $-40$  dBm/Hz. The sample rate  $F_s$  is 2.208 MHz. The equalizer size  $T$  is 16 and the initial values are set to  $\mathbf{v}_i^{(0)} = [0, \dots, 0, 1, 0, \dots, 0]^T$  with "1" in the  $(T/2)$ th position, and  $\mathbf{L}_i^{(0)} = 10^5 \cdot \mathbf{I}_T$ . PTEQ is compared with per 11 tones equalization for the RLS algorithm with inverse updating.

Fig. 3(a) shows the bitrate as a function of the number of training symbols for per tone (in dashed-dotted line) and per 11 tones (in solid line) equalization. Also the optimum bitrate, computed with direct initialization, is plotted for both schemes. The obtained bitrates are seen to be almost the same: 7.1882 Mbit/s (for per tone equalization, in dotted line) and 7.1719 Mbit/s (for per 11 tones equalization, in dashed line). In

<sup>3</sup>A multiplication of a real number with a complex number is counted as two real multiplications and a multiplication of two complex numbers is counted as four real multiplications.

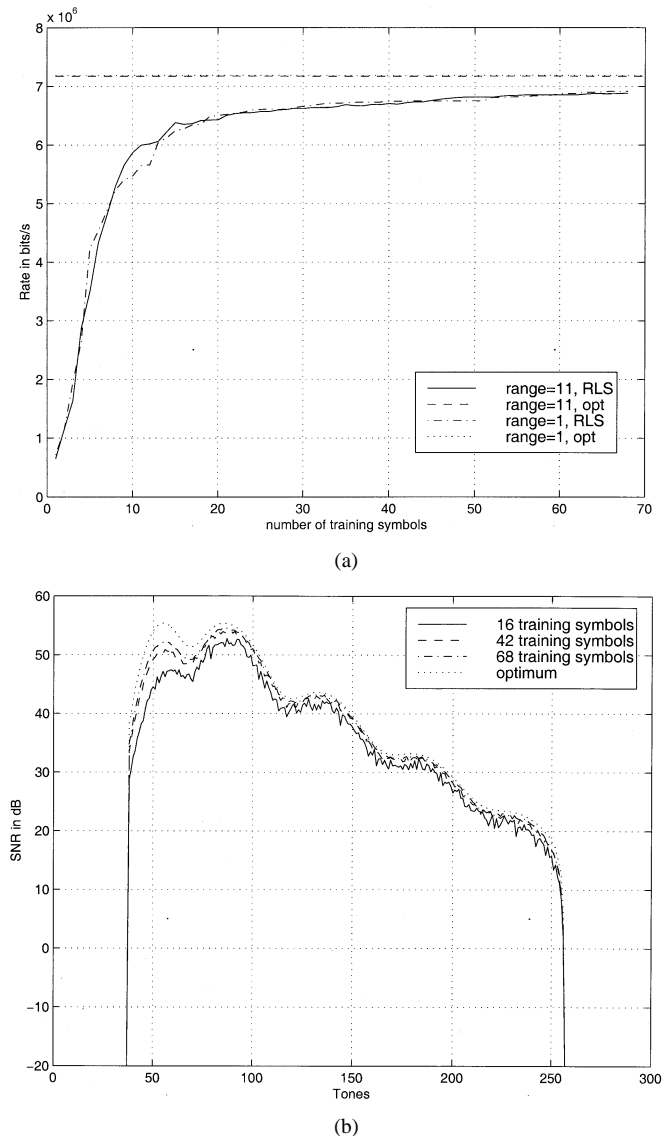


Fig. 3. Simulation results for RLS with inverse updating. (a) Bit rate for per tone and per 11 tones equalization. (b) SNR for PTEQ.

Fig. 3(b), SNR distributions are plotted for PTEQ for different lengths of the training sequence. It is seen that convergence to optimal performance may be achieved with an acceptably short training sequence.

#### VI. CONCLUSIONS

A recursive initialization procedure based on RLS with inverse updating is presented for PTEQ initialization. It is demonstrated that part of the signal flow graph is common to all used tones, leading to a significant complexity reduction. Complexity calculations are presented and simulation results show convergence with an acceptable number of training symbols.

#### REFERENCES

- [1] J. A. C. Bingham, "Multicarrier modulation for data transmission: An idea whose time has come," *IEEE Commun. Mag.*, vol. 28, pp. 5-14, May 1990.
- [2] S. Haykin, *Adaptive Filter Theory*, 2nd ed. Englewood Cliffs, NJ: Prentice-Hall, 1991.

- [3] P. J. W. Melsa, R. C. Younce, and C. E. Rohrs, "Impulse response shortening for discrete multitone transceivers," *IEEE Trans. Commun.*, vol. 44, pp. 1662–1672, Dec. 1996.
- [4] C. T. Pan and R. J. Plemmons, "Least squares modifications with inverse factorization: Parallel implications," *J. Comput. Appl. Math.*, vol. 27, pp. 109–127, 1989.
- [5] T. Pollet, H. Steendam, and M. Moeneclaey, "Performance degradation of multicarrier systems caused by an insufficient guard interval duration," in *Proc. Int. Workshop on Copper Wire Access Systems*, Budapest, Hungary, Oct. 1997, pp. 265–270.
- [6] K. Van Acker, G. Leus, M. Moonen, O. van de Wiel, and T. Pollet, "Per tone equalization for DMT-based systems," *IEEE Trans. Commun.*, vol. 49, pp. 109–119, Jan. 2001.
- [7] K. Van Acker, G. Leus, M. Moonen, and T. Pollet, "Frequency domain equalization with tone grouping in DMT/ADSL-receivers," in *Proc. Asilomar '99*, Pacific Grove, CA, Oct. 1999.
- [8] M. Van Bladel and M. Moeneclaey, "Time-domain equalization for multicarrier communication," in *Proc. IEEE Global Telecommunications Conf.*, Singapore, Nov. 1995, pp. 167–171.